MAR 22 1929

VOLUME LXIX NUMBER 1

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

EDWIN B. FROST
Yerkes Observatory of the University of Chicago

HENRY G. GALE
Ryerson Physical Laboratory of the University of Chicago

JANUARY 1929

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

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The Astrophysical Journal is published by the University of Chicago at the University of Chicago Press, 5750 Ellis Avenue, Chicago, Illinois, during each month except February and August. ¶ The subscription price is $6.00 a year; the price of single copies is 75 cents. Orders for service of less than a half-year will be charged at the single-copy rate. ¶ Postage is prepaid by the publishers on all orders from the United States, Mexico, Cuba, Porto Rico, Panama Canal Zone, Republic of Panama, Dominican Republic, Canary Islands, El Salvador, Argentina, Bolivia, Brazil, Columbia, Chile, Costa Rica, Ecuador, Guatemala, Honduras, Nicaragua, Peru, Hayti, Uruguay, Paraguay, Hawaiian Islands, Philippine Islands, Guam, Samoan Islands, Balearic Islands, Spain, and Venezuela. ¶ Postage is charged extra as follows: for Canada and Newfoundland, 30 cents on annual subscriptions (total $6.30); on single copies, 3 cents (total 78 cents); for all other countries in the Postal Union, 50 cents on annual subscriptions (total $6.50), on single copies 5 cents (total 80 cents). ¶ Patrons are requested to make all remittances payable to The University of Chicago Press, in postal or express money orders or bank drafts.

The following are authorized agents:

For the British Empire, except North America, India, and Australasia: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12*s.* 6*d.* each; single copies, including postage, 4*s.* each.

For Japan: The Marusen Company, Ltd., Tokyo.

For China: The Commercial Press, Ltd., Paoshon Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 50 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "Observatory, Williamsbay, Wisconsin."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Post-office at Chicago, Ill., under the act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October 3, 1917, authorized on July 15, 1918.

PRINTED IN THE U.S.A.

THE ASTROPHYSICAL JOURNAL

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

—

THE CAMBRIDGE UNIVERSITY PRESS
LONDON

THE MARUZEN COMPANY, LIMITED
TOKYO

THE COMMERCIAL PRESS, LIMITED
SHANGHAI

THE ASTROPHYSICAL JOURNAL

An International Review of Spectroscopy and Astronomical Physics

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VOLUME LXIX

JANUARY–JUNE 1929

THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS

72

Published
January, March, April, May, June, 1929

Composed and Printed By
The University of Chicago Press
Chicago, Illinois, U.S.A.

Astron. Obs.
Wash.

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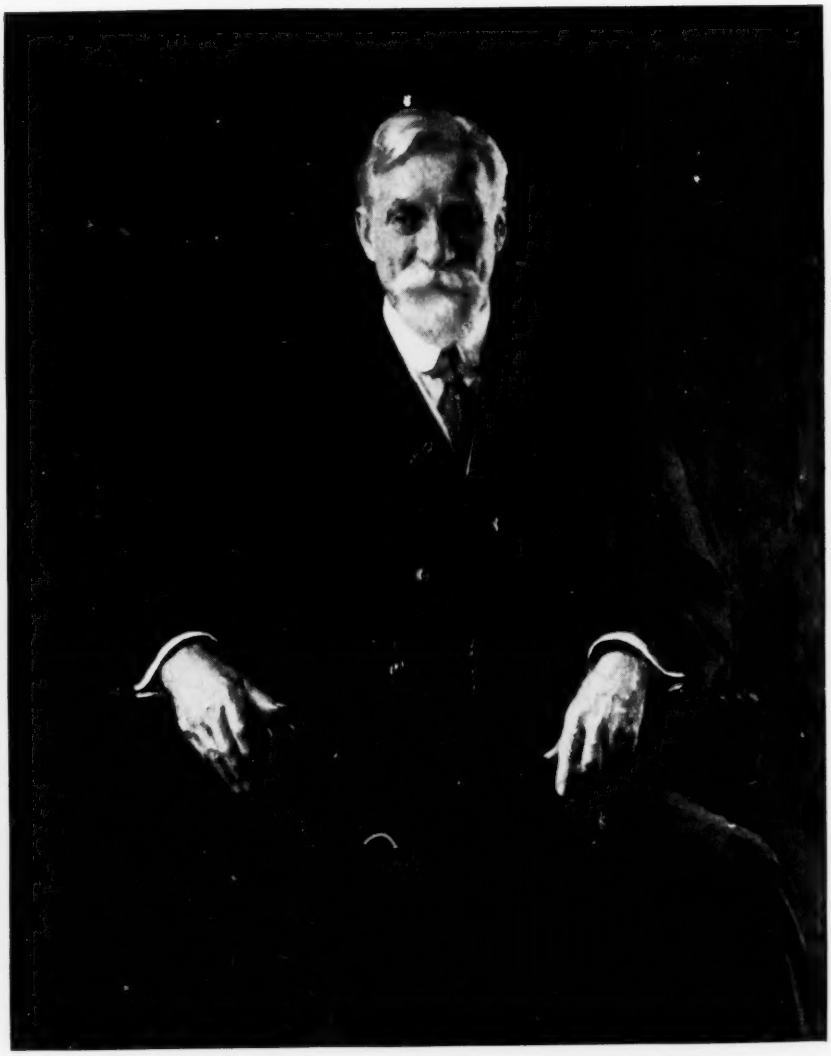

From the Painting by Ralph Clarkson

THOMAS CHROWDER CHAMBERLIN

1843–1928

THE ASTROPHYSICAL JOURNAL

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VOLUME LXIX JANUARY 1929 NUMBER 1

THOMAS CHROWDER CHAMBERLIN

By W. D. MacMILLAN

On November 15, 1928, the name of Thomas Chrowder Chamberlin was removed from the roll of active scientists by the hand of death. For three-score years he toiled diligently in the fields of geology and cosmogony. He served as president of the University of Wisconsin from 1887 to 1892, and did not allow the exacting duties of the presidency to prevent him from teaching a class in geology and maintaining his active interest in research. His efforts brought the institution from the status of a college into that of a university. To him deservedly came all the honors which his fellow-scientists could bestow. He was president of the Geological Society in 1894, president of the Chicago Academy of Sciences from 1898 to 1914, president of the Illinois Academy of Science in 1907, and president of the American Association for the Advancement of Science in 1907. He was a member of the National Academy of Sciences, the American Academy of Arts and Sciences, the American Philosophical Society, and many other societies in this country and abroad.

Although he was born in Illinois, he was but three years of age when his father, who was a Methodist circuit-rider and a farmer, moved to a farm a few miles north of Beloit, Wisconsin. There the lad grew to manhood, and it was from Beloit College that he received his Bachelor's degree (1866). His early ideals and ambitions were all for education, and after his graduation he became principal

of the high school at Delavan, Wisconsin (1866–1868). The eager curiosity of his pupils with respect to the stones, minerals, and fossils of the immediate neighborhood turned his attention to the study of geology. Feeling the need of a broader understanding of these matters on his own part, he resigned his position in order that he might spend a year at the University of Michigan in the study of Science. This preparation was sufficient to procure for him a position as professor of the natural sciences at the State Normal School at Whitewater, Wisconsin. Professor Chamberlin humorously referred to this position not as a "chair," but as merely a "settee." At any rate, after four years on the settee he was offered a real chair as professor of geology at his alma mater, Beloit College. He continued there as professor for nine years and as non-resident lecturer for five years more, and from 1885 to 1887 was a professor at the Columbian University at Washington. From 1873 to 1876 he was also assistant state geologist and thereafter chief geologist. From 1876 to 1882 he was the director of the Wisconsin State Geological Survey. It was inevitable from the character of the Wisconsin deposits that he should become a glacialist, and when he laid down the work of the State Geological Survey he was undoubtedly the leading American authority on glaciation. He resigned as president of the University of Wisconsin in 1892 in order that he might accept the position of Head of the Department of Geology at the new University of Chicago, a position which he continued to hold until his retirement in 1919 at the age of seventy-six.

In the Introduction to his little volume *The Origin of the Earth* (1916) Professor Chamberlin relates how his efforts to interpret the glacial deposits of Wisconsin led to the study of cosmogony.

> My early geologic work happened to fall in a tract that was overlain by a thick mantle of glacial drift and underlain by the sheeted sediments of the Paleozoic seas. Above, there was little but the products of the strange ice invasion; below, there was little within reach but the products of the ancient seas. . . . . Early bias favored the sea life, but the glacial beds were uppermost in the field and soon came to be foremost in the zest of inquiry. How ice sheets could have crept so far south upon the low plains in the heart of our continent grew to be a more and more insistent puzzle as the verity of the invasion grew more and more incontestable. There were, indeed, inherited theories, but when these were brought to the test by the precise realities of the record as it was met from day

to day in the field, they seemed to limp under the burden of explanation they had taken upon themselves, and so, one after another, they were turned out to pasture as lame horses no longer fit to be ridden. . . . .

It was generally held, in accordance with Laplace's beautiful theory of the origin of the solar system, that the earth was at first all gas—all atmosphere, as Laplace put it—and that all of the carbon later locked up in the coals, the oils, the carbonaceous shales, the limestones, and the other carbonates was then free gas and diffused throughout this great globe of gas. It was held that later, when cooling had made some progress, the refractory matter that was soon to form the rocks gathered into a white hot globe of lava, but that still all the oxides of carbon and all the water remained in the atmosphere and gave it enormous extent and density. . . . . And so, each earlier age was thought to have held more carbon dioxide in its atmosphere than the succeeding ones. If this were true, it seemed idle to go backward in time to find deficiency in carbon dioxide.

Beside this infelicity, there seemed to be in this very fact of a great decline from the very hot to the cold a basis for a plausible hypothesis of glaciation—the simple, natural trend of a moribund earth toward a cold senility. . . . . Our recent icy stage was but an October frost; December was yet to come. . . . .

But this theory of a simple decline from a fiery origin to a frigid end, from a thick blanket of warm air to a thin sheet of cold nitrogen, consonant with the current cosmogony as it was, logical under the premises postulated, pessimistically attractive in its grewsome forecast, already in possession of the stage, with a good prospect of holding it—this theory of a stupendous descensus none the less encountered some ugly facts as the inquiry went on. It seemed to accord well enough with an ice age, *if* the ice age came *only* in the later stages of the earth's history, but it was ill suited to explain an ice age in the earlier geologic eras. Unfortunately for it, there began to appear signs of ice ages far back in time, and besides some of these had their seats much nearer the equator, and, in other respects, were even stranger than the latest great glaciation. . . . . While all of this should have weakened, and did weaken, the fundamental concept of great warmth and a rich atmosphere in the earlier ages, while it should have roused skepticism as to the verity of the cosmogony on which it was based, and perhaps did so, still the old cosmogony continued to hamper all attempts at a radical revision of glacial theories. . . . .

Desiccation products were found to be scarcely less abundant and characteristic in the early strata than in the later, and no steady progress from humidity to aridity seemed to mark the progress of time; nor were there found any conclusive evidences of even an oscillatory progress from predominant humidity to predominant aridity. If the record favored any generalization, it seemed to be that the severest and most prevalent period of aridity fell near the middle of the stratigraphic record. . . . .

When the inquiry was pressed still farther back, and support for the postulate of a molten globe was sought in the crust itself, it was not forthcoming.

With strange perversity the supposed granite *foundations* proved to be granitic *intrusions*. Thus in a very literal sense the very foundations of the old view proved illusive.

It was thus that the trail was followed back, with a weakening faith in the inherited theory, till the borders of the primitive stage were reached. But one further step remained—to examine the cosmogonic postulates themselves. Could the earth ever have had the vast hot atmosphere postulated? Was the earth's gravity sufficient to hold so vast and vaporous an envelope at such high temperatures and in such an intense state of molecular activity as the old mode of genesis assigned? Was the gaseo-molten genesis a reality? Thus I was already across the pass that leads from the land of rocks into a realm of cosmogonic bogs and fens. Its mists were already gathering over the path ahead. Strangely enough, the cold trail of the ice invasion had led by this long and devious path into the nebulous field of genesis.

This story of the manner in which Professor Chamberlin was led into the fields of cosmogony is of intense interest to the astronomers. Hitherto cosmogony had been the province of the mathematical astronomers alone. The uniformity of the motions of the solar system contained implications of a dynamic character which had appealed to Laplace, and the contributions of Helmholtz and Lane had supported the interpretation which Laplace had offered, so that astronomers were quite satisfied with the view which was thus presented. Both the geologists and the biologists had long expressed dissatisfaction with the time-intervals which were allowable from this point of view, but to the point of view itself they had offered no objection; all that they asked for was more time.

To Dr. Chamberlin it was not merely the Helmholtzian time-scale that offered difficulty. The fundamental conclusions of a geological character which followed from the Laplacian hypothesis were at variance with the testimony of the rocks, and he frankly faced the difficulty. If hypotheses of a special creation are ignored, the origin of the earth is evidently an astronomical affair, and other possibilities of an astronomical character were sought. It seemed to Chamberlin that such a possibility could be found in the dynamical consequences of the close approach of another star to our sun. Not being a mathematician, it was but natural that he was unable to trace out these consequences, but he was extremely fortunate in finding in Dr. F. R. Moulton a collaborator who not only possessed

the high degree of technical skill which the problem required, but who also had the imagination and the sound philosophy necessary to make that collaboration fruitful.

The result of this collaboration was the planetesimal hypothesis of the origin of the sun's family of planets. It has little, perhaps nothing, in common with the nebular hypothesis of Laplace. It coordinates the uniformities of motion of the solar system quite as simply as did the hypothesis of Laplace, and it avoids many of the difficulties which the older hypothesis had encountered without introducing new ones of its own. Furthermore, and this is a point which mathematical astronomers are likely to overlook or at least underestimate, it furnished a foundation for the geologists which is in harmony with the evidences of their own science. This is not to say that everything is perfectly clear. Many problems remain, both of an astronomical and geological character, but apparently there is no evidence in either domain which is of a distinctly contradictory character.

Dr. Chamberlin always favored the method of multiple hypotheses. Every hypothesis, even his own, was held tentatively. Other hypotheses, when such were available, were kept in mind and weighted according to the evidence. To him it seemed that the planetesimal hypothesis was more probable than others, but he was always ready to listen to a reasonable argument about it. Many astronomers and geologists of the present generation regard it with favor, but whatever fate the future may hold for it, students of cosmogony will ever be grateful to Dr. Chamberlin for his splendid contribution to their domain. His researches were in part conducted under the auspices of the Carnegie Institution of Washington, from 1902 onward, and after 1909 as research associate of the institution.

In January, 1893, Professor Chamberlin issued, as editor-in-chief, the first number of the *Journal of Geology*, under the auspices of the University of Chicago, and for thirty-six years he maintained this heavy responsibility, which has had a marked effect upon the research of fundamental problems in this branch of science. In this *Journal* appeared most of Professor Chamberlin's contributions, in the form of articles, during this period. Among his major works,

outside of the publications of the geological surveys, were his textbook on *Geology*, Volume I (1904); second edition (1905); Volume II (1905); Volume III (1906), jointly written with the late R. T. Salisbury; *The Tidal and Other Problems*, "Publications of the Carnegie Institution of Washington" (1909), including articles by himself and five associates whose researches were inspired by him. Smaller volumes of much interest are: *The Origin of the Earth* (1916); and *The Two Solar Families*, which came off the press only a fortnight before the death of its distinguished author.

UNIVERSITY OF CHICAGO

PHYSICAL PROPERTIES OF A GASEOUS SUBSTRATUM IN THE GALAXY

By B. P. GERASIMOVIČ and O. STRUVE

ABSTRACT

By means of the *observed intensities* of the detached Ca^+ lines in stars of various apparent magnitudes, and by means of the corresponding *distances*, the average *absorption coefficient per centimeter* was found to be $\beta = 1.1 \times 10^{-22}$.

Taking into consideration the *probabilities of different electronic transitions* for the Ca^+ atom, the interstellar *density of* Ca^+ is $\rho_{Ca^+} = 3.6 \times 10^{-32}$.

In order to compute the actual density of the interstellar gas, consisting of atoms of *many elements in various states of ionization*, the ionizing efficiency of the diluted radiation from all stars in space was computed. This yielded the relative concentration of *Ca* atoms in different states of ionization. Making reasonable assumptions concerning the *partial abundance of Ca* in the gas, the total *density* turns out to be *of the order of* 10^{-26}.

These results were *applied to Na*, but the computed intensities relatively to the neighboring portion of the continuous spectrum were found to be too small, which might be accounted for by inaccuracy of the adopted value of the probability coefficients for D lines.

The *effect of nearby* stars upon the general absorption was investigated but found to be *negligible*. Observational evidence does not favor the existence of condensations near the stars.

The hypothesis of *an interstellar substratum embodying the whole galactic system* is the most satisfactory at present. This hypothetical substratum *shares the rotational motion* of the stars around a distant central mass in galactic longitude 325°.

When A. S. Eddington[1] discussed the hypothesis that interstellar space is filled by a tenuous cloud of atoms, and that it is this cloud which is responsible for the appearance of detached lines in stellar spectra, observational evidence was limited to J. S. Plaskett's[2] demonstration that the radial velocities from such detached lines are independent of the motions of the stars themselves, and that they are almost (but not quite) identical with the reflex of the solar motion. Since that time additional observational material has come to hand;[3] not only have the conclusions of Plaskett regarding the radial velocities been confirmed, but information has become available concerning the intensities of the detached lines in a large num-

[1] *The Internal Constitution of the Stars*, p. 371, 1926; *Proceedings of the Royal Society of London*, A, **111**, 424, 1926.

[2] *Publications of the Dominion Astrophysical Observatory, Victoria*, **2**, 335, 1924; *Monthly Notices of the Royal Astronomical Society*, **84**, 80, 1924.

[3] O. Struve, *Contributions from the Mount Wilson Observatory*, No. 331, 1927; *Astrophysical Journal*, **67**, 353, 1928.

ber of stars. The observational side of the question, including the degree of precision attainable in making estimates of such intensities, was recently discussed in this *Journal*.[1] It seemed desirable to extend the investigation to include the physical aspects of the whole problem. It was our intention to derive from the observational data all possible information concerning the density and distribution of diffuse matter in interstellar space, and to investigate the resulting physical properties of this hypothetical gaseous substratum of our galaxy. It will be seen that, aside from one or two minor difficulties, the results are in excellent harmony with all modern ideas concerning the structure of the universe.

I. THE ABSORPTION COEFFICIENT OF THE INTERSTELLAR CALCIUM LINE K

Let I be the intensity of the dimmed light and I_0 that of the undimmed. We have

$$I = I_0 \cdot e^{-\beta/\pi} \tag{1}$$

where β is the average effective absorption coefficient per parsec and π is the parallax. The quantity that was actually observed was

$$\Delta m = -2.5 \log I/I_0 = 2.5 \bmod \beta/\pi \ .$$

Let us calculate the mean value of Δm for stars of the same apparent brightness and spectral subtype. It is known[2] that if the log of a statistical function is a polynomial of the second degree, the logs of the parallaxes of stars of the same apparent brightness have a normal distribution around log π_m° with a probable error r, where π_m° is the most probable parallax for stars of apparent magnitude m.

$$\pi_m^\circ = \bar{\pi}_m \cdot e^{-5.827 r^2}.$$

$\bar{\pi}_m$ is the mean parallax for the same stars and

$$r = 0.6745 \sqrt{\frac{P_1}{10 h^2}} \ ,$$

[1] *Loc. cit.*

[2] J. C. Kapteyn, *Publications of the Astronomical Laboratory at Groningen*, **29**, 43, 1918.

where h is the modulus of precision for the normal luminosity curve and P_1 is a coefficient in a known expression

$$\log \bar{\pi}_m = P_0 - P_1 \cdot m \ .$$

To obtain a mean value of Δm, i.e., $\overline{\Delta m}$ for stars of the same apparent magnitude and spectrum, we calculate

$$\overline{\Delta m} = \frac{1.086\beta \int_{-\infty}^{+\infty} e^{-\frac{(0.6745)^2}{2r^2}[\log \pi - \log \pi_m^\circ]^2} \frac{1}{\pi} d \log \pi}{\int_{-\infty}^{+\infty} e^{-\frac{(0.6745)^2}{2r^2}[\log \pi - \log \pi_m^\circ]^2} d \log \pi}$$

$$= \frac{1.086\beta e^{\frac{r^2\left(\frac{1}{\text{mod}} - \frac{(0.6745)^2}{r^2} \log \pi_m^\circ\right)^2}{2(0.6745)^2}}}{e^{\log^2 \pi_m^\circ \cdot \frac{[0.6745]^2}{2r^2}}} ,$$

or finally

$$\overline{\Delta m} = \frac{1.086}{\pi_m^\circ} e^{5.827 r^2} \beta = \frac{1.086}{\bar{\pi}_m} e^{11.654 r^2} \beta \ .$$

In a recent paper[1] the following relationships were derived from all available proper motions and radial velocities:

For Bo–B2 stars: $\log \bar{\pi}_m = -2.474 - 0.050m$; $r = 0.32$
For B3 stars: $\log \bar{\pi}_m = -1.870 - 0.097m$; $r = 0.094$

The value of r for Bo–B2 stars is not very trustworthy since for these stars the derived value h is evidently too small. Unquestionably the dispersion of absolute magnitudes increases rapidly as we approach the early B's,[2] but less rapidly than the foregoing relationships would indicate (for B3, $h = 0.070$; for Bo–B2, $h = 0.15$). We shall therefore calculate with two values for Bo–B2, $r = 0.32$ and $r = 0.16$.

From the estimates of the intensities of the K line in the spectra of 1623 Bo–B3 stars,[3] we can deduce β for various apparent magnitudes. For convenience all stars were divided into two groups, viz.,

[1] Gerasimovič, *Vierteljahrsschrift der Astronomischen Gesellschaft*, **61**, 219, 1926.

[2] O. Struve, *Astronomische Nachrichten*, **231**, 17, 1927.

[3] O. Struve, *Astrophysical Journal*, **67**, 353, 1928.

Bo–B2 and B3. Accordingly, there are for each apparent magnitude two equations of condition of the form

$$\overline{\Delta m}+C=\beta\frac{1.086\cdot e^{11.654r}}{\pi_m}.$$

C is a correction due to the possible influence of a stellar K line. We have seen[1] that for subdivision B3, at least, this cannot be neglected.

A preliminary graphical solution gives

For Bo–B2: $C=+0^m.01$
For B3: $C=-0^m.12$

This agrees well with what might have been anticipated from the general run of K-intensities in the later subdivisions.[2]

Table I gives the results of our computations for β. As was stated before, we have used Gerasimovič's value $r=0.094$ for B3, while we have used $r=0.32$ and $r=0.16$ for Bo–B2.

From Table I we derive the following conclusions: (1) The absorption coefficient does not depend upon distance. The small increase of β up to 5.5–6.5 mag. might be explained by errors in the adopted parallax formulae, or by small systematic differences in the observed Ca^+ intensities. (2) The computed values of β for stars of the same spectrum agree surprisingly well. (3) The dispersion of the absolute magnitudes for Bo–B2 is not as great as the adopted parallax formula shows. Best agreement with B3 is obtained by adopting for the modulus of precision, h, a somewhat larger value than 0.15, about 0.30. The corresponding value of r is 0.16. Column 3 of Table I computed with this value gives an excellent agreement with B3.

In view of the fact that our parallax formula for B3 is the more reliable one, we shall adopt in all further computations

$$\beta=3.4\times10^{-4}.$$

In our formulae β denotes an average absorption coefficient for the average distance from the Sun to stars of every given apparent magnitude. Therefore, if β is constant, the true absorption coefficient in space will be also constant. This conclusion seems safe unless our

[1] *Loc. cit.* [2] *Ibid.*

computations are affected by systematic errors. These can be of two kinds: (1) errors in the spectrophotometric estimates, depending upon apparent magnitude. This source of error was discussed previously,[1] and it is believed that our observations are not materially affected by it; (2) errors in the parallaxes of the fainter B-type stars, depending upon systematic errors of the proper motions. Although this kind of error is quite possible, the following arguments are against it. The computed parallaxes for bright and faint B stars were tested by calculating mean space velocities from the tangential

TABLE I

AVERAGE ABSORPTION COEFFICIENT FOR INTERSTELLAR K PER PARSEC

MEAN APPAR. MAG.	B0–B2		B3
	$r=0.32$	$r=0.16$	$r=0.094$
2.5	1.1×10^{-4}	2.8×10^{-4}	1.3×10^{-4}
3.5	1.1	2.8	3.0
4.5	1.7	4.2	3.6
5.5	2.0	5.0	4.2
6.5	1.6	4.0	4.4
7.5	1.6	4.0	4.1
8.5	1.4	3.5	3.6
9.5	1.3	3.2	2.9
10.5	1.3×10^{-4}	3.2×10^{-4}	3.7×10^{-4}
Mean	1.4×10^{-4}	3.6×10^{-4}	3.4×10^{-4}

component and from radial velocity separately. The good agreement between the resulting space velocities seems to exclude the possibility of large systematic errors in the parallaxes of the faint B stars as deduced from proper-motion data.[2] On the other hand, it seems quite improbable that the error under investigation would exactly compensate the real variation of β in two independent groups: B0–B2 and B3.

There is a little difficulty in reconciling the constancy of β with the results in Figure 6, *Astrophysical Journal*, **67**, 387, 1928. It was suggested there that the density of *Ca* atoms decreases with distance. However, that result depended entirely upon the assumed mean dis-

[1] Struve, *Astrophysical Journal*, **67**, 358, 1928.

[2] Gerasimovič, *op. cit.*, **61**, 224, 1926.

tance of the P Cygni stars, $D=6500$ parsecs. If this distance is reduced to about $D=3000$ parsecs, a linear relationship for $I=f(D)$ is obtained which would correspond to a constant β. While there is no direct evidence concerning the distances of the P Cygni stars, the linear relation has the advantage of giving more plausible distances for three novae than would result from our former curve. All other distances remain practically the same. We are thus inclined to regard β as constant within the limits reached by our observations.

In what follows we shall adopt for the true absorption coefficient (per cm) for the interstellar K line

$$1.1\times10^{-22}.$$

II. THE DENSITY OF Ca^+

The total absorption of a slab of gas for a given line[1] is

$$\int \alpha_\nu d\nu = \frac{h\cdot\nu}{c}[B_{ij}N_i - B_{ji}N_j] ;$$

α_ν is the absorption coefficient between ν and $\nu+d\nu$, and the integration is performed along the width of the line. The B's are Einstein's transition coefficients for two states j and i $(j>i)$ connected by the well-known relations:

$$A_{ji}=\frac{p_i}{p_j}\frac{8\pi h\nu^3}{c^3}B_{ij}=\frac{8\pi h\nu^3}{c^3}B_{ji} ; \qquad (2)$$

p_i and p_j are the a priori weights of the given states. Let us consider two levels of the Ca^+ atoms, the normal level 1σ $(i=1)$ and the excited $1\pi(j=3)$. We shall regard the $1\pi-1\sigma$ doublet (K and H) as a single line. Let us assume

$$N_1=N_+n_1 \text{ and } N_3=N_+n_3 ,$$

where N_+ is the total number of Ca^+ atoms and n_i is the partial concentration in the i state. The number n_3 will be very small in comparison with n_1 when the exciting radiation is greatly diluted. Let us at first suppose that we have a detailed balancing of elementary processes similar to that encountered in thermodynamic equilibrium. Then

$$n_3[A_{31}+B_{31}u(\nu)]=n_1\cdot B_{13}\cdot u(\nu) ,$$

[1] R. C. Tolman, *Statistical Mechanics*, p. 175, 1927.

where $u(\nu)$ is Planck's function for radiation of frequency ν multiplied by a very small dilution factor W. Taking into account Einstein's relations (2) we get

$$n_3/n_1 = p_3/p_1 \cdot \frac{u'(\nu)}{1+u'(\nu)} ,$$

where $u'(\nu) = u(\nu) : 8\pi h\nu^3/c^3$. For the case under consideration $u'(\nu)$ is extremely small, because, as we shall see, W is of the order of 10^{-17}. Accordingly, n_3/n_1 is a negligibly small quantity. Of course, it is true that interstellar calcium is in a condition very far removed from thermodynamic equilibrium. We have no detailed balancing, and we have to take into consideration ionizations and photo-electric captures from both states, as well as the influence of the 1δ states intermediate between 1σ and 1π. This will change the value of n_3/n_1 as derived above on the principle of detailed balancing, but it is obvious that the more accurate method would give about the same order of magnitude of n_3/n_1. We have

$$\int a_\nu d\nu = \frac{h\nu N_+ n_1}{c}\left[\frac{p_3}{p_1} - \frac{n_3}{n_1}\right] B_{31} = \frac{c^2 A_{31}}{8\pi\nu^2}\,\frac{p_3}{p_1} N_+ .$$

This is the formula used by Milne in his theory of the Ca^+ chromosphere. From the chromosphere we obtain the value of A_{31}. For the combined $1\pi_1$ and $1\pi_2$ states $p_3/p_1 = 3$, and according to Milne,[1]

$$(A_{31})^{-1} = 1.8 \times 10^{-8}\left(1 + \frac{I_{1R}}{I_{HK}}\right) ,$$

where I_{1R} and I_{HK} are the combined intensities of the infra-red doublet with satellite (X) and of the principal doublet (H and K) in the solar chromosphere. According to recent data by Davidson,[2] we can take $I_{1R}/I_{HK} = 1.6$ for the lower chromosphere. This gives

$$A_{31} = 2.1 \times 10^7 .$$

For each line $\int a_\nu d\nu = \beta\Delta\nu$, where $\Delta\nu$ is the frequency interval corresponding to the width of the line. Next we take into account the H line, since the foregoing value of A_{31} relates to the combined 1π states. The theoretical values of the intensities of emission for

[1] *Monthly Notices of the Royal Astronomical Society*, **86**, 9, 15, 24, 1925.

[2] *Ibid.*, **88**, 30, 1927.

the K and H lines, confirmed by measures in the laboratory, are as 2 to 1.[1] The same values should be approximately true in absorption. Accordingly, we use for the K line[2] $A_{31}=6\cdot3\times10^{7}$, per 1 cm³.

On the other hand, we have

$$\beta\Delta\nu=\int a_\nu d\nu=1.1\times10^{-22}\Delta\nu\,,$$

where $\Delta\nu$ relates to the cosmic K line.

We are now able to calculate N_+, i.e., the number of cosmic Ca^+ atoms per cm³. Taking into account that $\Delta\nu=3\times10^{18}\Delta\lambda/\lambda^2$, where λ is expressed in angstroms, we get

$$N_+=1.8\times10^{-9}\Delta\lambda\,.$$

The mass of a Ca atom being 6.6×10^{-23}, the density of interstellar Ca^+ will be

$$\rho_{Ca^+}=1.2\times10^{-31}\Delta\lambda\,.$$

Concerning the true width of the cosmic K line, $\Delta\lambda$, we know but little. Direct measurements on one-prism spectrograms taken at the Yerkes Observatory give $\Delta\lambda=0.4$ A. But the estimates of intensity refer to the center of the line, while β in formula (1) is, of course, the average absorption coefficient over the whole line. It is necessary either to use average residual intensities for K or to assume for $\Delta\lambda$ a correspondingly smaller value. Taking into account the contour of the line, we obtain for the effective width of K, $\Delta\lambda=0.3$ A.

It should be noted that the measured value of 0.4 A is exactly equal to the minimum width imposed by the degree of purity of this spectrograph. Strictly speaking, this value is only an upper limit for the true width: a spectrograph with greater resolving power would probably yield a smaller value of $\Delta\lambda$. However, the uncertainty does not affect the value of N_+ nor, consequently, that of ρ_{Ca^+}. Suppose, for example, that the true value of $\Delta\lambda$ is not 0.3 but $0.3/r$ A. In that case the measured intensity does not represent the

[1] Dorgelo, *Zeitschrift für Physik*, **31**, 827, 1925.

[2] By definition, the "life" of an atom:

$$\tau=\tau_1+\tau_2=\frac{1}{A_{13}}=\frac{1}{A_K}+\frac{1}{A_H}=\frac{1}{A_K}\left(1+\frac{A_K}{A_H}\right)=\frac{1}{A_K}\left(1+\frac{I_K}{I_H}\right)=\frac{1}{A_K}\times3\,.$$

true central intensity and, consequently, the true value of β will be larger. From experimental data we may assume that a very narrow line in spectrographs of varying degrees of purity preserves $\Delta\lambda \cdot \Delta m = \text{const.}$ In other words, the total absorption remains unaltered. Since $\Delta m \sim \beta$ we may assume that $\Delta\lambda \cdot \beta = \text{const.}$ It is this product which enters our formula for N_+, and our factor of uncertainty r cancels in the final result.

Substituting $\Delta\lambda = 0.3$ A in equation (3), we derive

$$\rho_{Ca^+} = 3.6 \times 10^{-32}.$$

III. THE DENSITY AND TEMPERATURE OF INTERSTELLAR RADIATION

The field of radiation in interstellar space produced by the integrated effect of all starlight is composed of contributions from a large number of sources. Accordingly, it does not correspond to simple black-body radiation; it has no definite temperature, and its density between ν and $\nu + d\nu$ cannot be represented by an expression of the type $W \cdot I(\nu T)d\nu$, where W is a constant dilution factor and $I(\nu T)$ is the density of black-body radiation corresponding to a constant temperature T and frequency ν. In our case the dilution factor W and the temperature T will be different for various parts of the spectrum, and their functional relationship to ν is probably very complicated. However, for a sufficiently small interval of ν we may regard both W and T as constant.

We are here concerned only with frequencies larger than those corresponding to 6.1 volts—this being the ionization potential of *Ca*. The quanta that are effective in the ionization of neutral *Ca* correspond to frequencies between 6.1 and 11.8 volts, those effective in ionizing Ca^+ correspond to values greater than 11.8 volts but probably not more than about 17 volts. For the frequency intervals just indicated we shall determine W and T. Since this determination necessarily depends upon knowledge of the distribution of the stars in space, it can be performed only for a point in space situated in the vicinity of the solar system. It is an unavoidable but doubtless permissible extrapolation to apply the same values of W and T for all points within our local system.

Let T_* be the effective temperature of a given star, W_* the corresponding dilution factor depending upon its size and distance, p_* the factor by which the energy must be multiplied in order to give luminous intensity. If a is the constant of Stefan's law, the total apparent density of radiation of all stars of a given spectral type, as observed visually, will be

$$\Sigma p_* a T_*^4 W_* = a p_i T_i^4 \Sigma W_* = a p_i T_i^4 W_i ,$$

where i is an index of spectral type; p_i is a known function of T_i[1] and $W_i = \Sigma W_*$ is to be determined from the observations.

Let Δ be the total density of the integrated starlight as observed visually, V_i the corresponding weight, in percentage, of a given spectral type. Then

$$a p_i T_i^4 W_i = \Delta \cdot V_i .$$

The observations give Δ and V_i. According to Van Rhijn,[2] the total amount of starlight over both hemispheres is equal to 1440 stars of 1.00 mag. This amount has been corrected for atmospheric absorption to the zenith. The maximum sensitiveness of the eye for low intensities (from the threshold to 0.00024-m candle) is at 0.503 μ.[3] The transmission coefficient for Mount Wilson, where the observations were made, is 0.86 for that wave-length,[4] consequently the resulting extra-terrestrial brightness corresponds to 1674 stars of 1.0 mag. Adopting the value of the solar constant as given by Abbot, the visual magnitude of the Sun as -26.5 according to Cerasky,[5] and $p = 1$ for the Sun, we calculate that the extra-terrestrial density of interstellar radiation, as observed visually, is $\Delta = 0.8 \times 10^{-13}$ erg/cm.[3] The foregoing involves no hypothesis as to the spectral constitution of this radiation. We now calculate the weights V_i for various spectral types. To do this with an error not exceeding 10 per cent it is necessary that we know the spectral types of faint stars at least up to 16 mag. According to Van Rhijn's data,[6] the total starlight per

[1] The function p_i is tabulated in Eddington's *Internal Constitution of the Stars*, p. 138, 1926.

[2] *Publications of the Astronomical Laboratory at Groningen*, **31**, 37, 1921.

[3] *Smithsonian Physical Tables*, p. 180, 1914. [4] *Ibid.*, p. 181.

[5] *Annales de l'Observatoire Astronomique de Moscou*, **5**, 30, 1911.

[6] *Op. cit.*, p. 72.

square degree for the galactic equator is 0.080 and for the galactic pole 0.010, the unit being a star of 1.0 mag. Fessenkoff and Staude[1] have found from Pannoekoek's data that the total light of all stars fainter than 7, 10, and 16 mag. is, per square degree:

Mag.	>7	>10	>16
Gal. lat. 0°	0.044	0.034	0.008
Gal. lat. 90°	0.007	0.003	0.001

Taking into account only stars up to 9 mag., we neglect 46 per cent of all starlight for the galactic equator and 40 per cent for the galactic pole; taking as limiting magnitude $m=12$, we neglect 32 per cent and 30 per cent, respectively. For stars up to 9 mag. we have complete data in the *Henry Draper Catalogue* from which we compute V_i by performing the numerical integration of the brightness of the *H.D.* stars. The results for V_i (expressed in percentages) based on Shapley's counts[2] up to 9.0 mag. are shown in the third column of Table II.

TABLE II

Spectral Type	T_i	V_i	W_i
B0–B5	17000	3.6	6.8×10^{-18}
B8–A3	11600	25.9	2.0×10^{-16}
A5–F3	7900	8.3	2.0×10^{-16}
F5–G3	6040	10.2	6.8×10^{-16}
G5–K3	4570	45.6	1.3×10^{-14}
K5–M	3430	6.4	1.2×10^{-14}

The inclusion of the stars of the partly published Henry Draper Extension would increase the accuracy of the results in an illusory manner only; it would diminish the region of extrapolation by 10 per cent, but at the expense of introducing a new systematic error produced by statistical selection. From the existing data on the spectra of faint stars we anticipate that no material changes in the deduced weights for the whole sky would be produced by the inclusion of all stars; the relative increase of faint A's in low galactic latitudes would be compensated by a large decrease of these stars

[1] *Russian Astronomical Journal*, **2**, Part III, 25, 1925.

[2] *Harvard College Observatory Circular*, No. 226, 1921.

in higher latitudes. There may be a relatively small increase in K's and decrease of B's. We can show indirectly that the fainter stars will not affect the relative values of V_i. If we take the average color indices for different spectral types[1] and weight them (as intensities) according to the values of V_i, we obtain for the weighted color index of all *Henry Draper Catalogue* stars +0.71, corresponding to spectral type G3. The observed integrated spectrum of the galactic star-clouds corresponds approximately to type Go. The agreement between the mean spectral type of the *Henry Draper Catalogue* stars and that of the galactic star-clouds is as good as could be expected. Consequently, our values for V_i represent sufficiently well the weights of the various spectral types for all stars contributing to the visible light of the galactic clouds. Table II contains all data used for the computation of the W_i. The temperature scale is that of E. S. King. For Bo–B5, Miss Payne's data were used. For the high frequencies with which we are dealing, Wien's approximation for the radiation law is quite sufficient. Therefore

$$W \cdot e^{-\frac{h\nu}{KT}} = \Sigma W_i e^{-\frac{h\nu}{KT_i}},$$

where W is the dilution factor and T the temperature of interstellar radiation for frequency ν.

W and T must be determined as functions of ν. The accompanying diagram shows the values of $\log We^{\frac{-h\nu}{KT}}$ for those ν's which are of greatest importance for the ionization of Ca ($h\nu = 6.1$) and of Ca^+ ($h\nu = 11.8$). Two straight-line solutions may be adopted, viz.:

$h\nu$ between 6 and 11 volts: $W = 1.5 \times 10^{-16}$; $T = 12{,}600°$
$h\nu$ between 11 and 17 volts: $W = 2.2 \times 10^{-17}$; $T = 15{,}300°$

The ionization potential of Ca^{++} has recently been determined by Bowen[2] as 51 volts. Computation shows that when $h\nu$ is between 50 and 60 volts the following values could be adopted: $W = 0.8 \times 10^{-17}$; $T = 16{,}900$. We know that the number of ionizations by quanta $h\nu$ decreases as $e^{\frac{-h(\nu-\nu_0)}{KT}}/h\nu$, where $h\nu_0$ is the ionization poten-

[1] *Annals of Harvard College Observatory*, **80**, 147, 1917.

[2] *Physical Review*, **31**, 502, 1928.

tial; thus the limits adopted above for our linear solutions (11–17, 50–60 volts) correspond closely to the most effective quanta.

It is noteworthy that when $h\nu$ is between 5 and 10 volts, W and T are chiefly determined by the radiation of A stars; when between 11 and 14 volts, the contribution of the B's becomes larger and for higher frequencies it becomes the most important. The contribution of the O's is negligible owing to their small number.

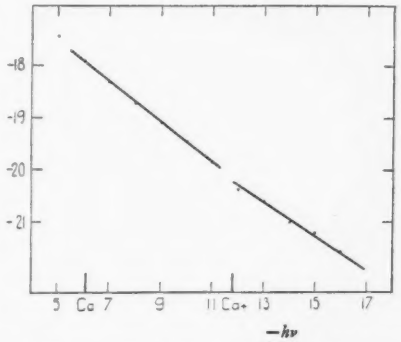

FIG. 1.—Graph of $\log We^{\frac{-h\nu}{KT}}$. The abscissae are $h\nu$ in volts.

We calculated W under the assumption that the *Ca* cloud does not produce any appreciable absorption of light beyond the limits of the principal series; good reasons for this supposition have been given by Eddington.[1]

IV. IONIZATION AND DENSITY OF INTERSTELLAR MATTER

The full formula for the ionization of a slab of gas in steady state and of finite optical thickness, stimulated by a diluted black-body radiation, has been deduced elsewhere.[2] If the slab is sufficiently transparent this formula becomes

$$\frac{x_{i+1}}{x_i} N_e \frac{h^3}{(2\pi m)^{\frac{3}{2}}(KT_0)^{\frac{1}{2}}KT} = We^{-\frac{h\nu_i}{KT}}, \qquad (3)$$

where x_i is the percentage of atoms in an ionization state i, T_0 the equivalent temperature of the electrons, i.e., a constant characterizing the Maxwellian distribution of electronic velocities, and N_e the number of free electrons per cm³. T_0 is a decreasing function of the optical thickness,[3] being equal to T in the limiting case of a transparent nebula (neglecting the squares and higher powers of

[1] *Proceedings of the Royal Society of London*, A, **111**, 440, 1926.

[2] Gerasimovič, *Proceedings of the American Academy of Arts and Sciences*, **62**, 164, 1927; *Harvard Reprint*, No. 38.

[3] *Ibid.*, p. 163.

$e^{-\frac{h\nu_i}{KT}}$). Two different factors are to be considered here: (1) During the captures the atoms pick up slow electrons, thus tending to raise the equivalent temperature T_0 (neglecting the hyperbolic hyperbolic transitions). (2) On the other hand, if the nebula is sufficiently untransparent, i.e., when its own radiation cannot be neglected (the latter being "cooler" than that of the stimulating star), the effective T and therefore the electron temperature T_0 will tend to diminish. Since the calcium cloud may be considered as transparent (except for line-absorption) we can put $T_0 = T$, where T relates to the most important ionization level (we shall see that in our case T corresponds to the transitions $Ca^+ \to Ca^{++}$).[1]

Let N be the total number of Ca atoms per cm^3 in all ionization states; we shall neglect for the present the Ca^{+++} atoms (we shall see later that we are justified in this). To determine x_0, x_1, x_2, N and N_e, we have two equations of the form (3) with $i=0$ and $i=1$, and the following:

$$N_e = N(x_1 + 2x_2)$$
$$Nx_1 = N_+ = \text{No. of } Ca^+ \text{ atoms determined in sec. II}$$
$$x_0 + x_1 + x_2 = 1$$

Preliminary calculation shows that practically all Ca atoms will be in Ca^{++} state; the number of Ca^{+++} atoms being more than seven thousand times smaller than that of Ca^+. Having in view the fact that for the reactions $Ca \to Ca^+$ and $Ca^+ \to Ca^{++}$ the values of W and T are nearly the same, we adopt in what follows $W = 2.2 \times 10^{-17}$ and $T = 15{,}300°$, as determined above for $Ca^+ \to Ca^{++}$ reactions.

In order to compute the total density of interstellar matter, we must take into account all other atoms besides Ca. It is certain that interstellar matter has a composite chemical structure, so that the free electrons are formed from the atoms of various elements. If p is the partial abundance of Ca atoms in interstellar space, we can put $pN_e = N(x_1 + 2x_2)$, assuming that the average efficiency of the atoms of various elements in producing free electrons is approxi-

[1] In his paper Eddington determined the lower limit of T_0 as $\frac{2}{3}T$; this is the "initial temperature" of the expelled electron, i.e., the temperature corresponding to the mean kinetic energy of projection, without taking into account the weeding-out of slow electrons during capture.

mately the same. The terrestrial abundance of *Ca* by weight is 3.4 per cent[1] and by atoms 1.5 per cent; in the latter case $p = 1.5 \times 10^{-2}$. The interstellar clouds are presumably formed during astronomical time by the permanent outflow of atoms from the stellar atmospheres, produced by selective radiation pressure. The interstellar partial abundance of *Ca* is therefore considerably larger than that in the earth's crust. We shall make two solutions: one with $p = 1$ and the other with $p = 1.5 \times 10^{-2}$. The true density of interstellar gas must be greater than the value derived from the first solution but smaller than that from the second.

TABLE III

IONIZATION AND DENSITY OF COSMIC GAS

	$p=1$	$p=1.5\times10^{-2}$
x_0	4×10^{-13}	3×10^{-11}
x_1	8×10^{-6}	7×10^{-5}
x_2	1	1
x_3	2×10^{-7}	2×10^{-8}
N_e	1×10^{-4}	1×10^{-3}
N_{Ca}	7×10^{-5}	7×10^{-6}
Total N	7×10^{-5}	5×10^{-4}
ρ_{Ca}	5×10^{-27}	5×10^{-28}
Total ρ	5×10^{-27}	1×10^{-26}

The total density of our cosmic cloud ought to be between 5×10^{-27} and 1×10^{-26}. A density of 10^{-24} was recently derived by Eddington[2] in an entirely different manner. Eddington starts from the well-known theory of isothermal gaseous spheres. He assumes the existence of local nebular condensations in space having a temperature of 10,000°; such nebulae, being isothermal, would extend indefinitely into space. Assuming that at a distance of 150 parsecs from the center the density of a local condensation is approximately equal to the density of interstellar gas in general, he obtains a value of 10^{-24}.

We have seen that only 10^{-11} of all *Ca* atoms are in the neutral state. This seems to exclude any possibility of detecting the *Ca* arc

[1] Clarke and Washington, *Proceedings of the National Academy of Sciences*, **8**, 108, 1922.

[2] *Proceedings of the Royal Society of London*, A, **111**, 428, 1926.

lines of interstellar origin. Of the lines of the Ca^+ atom, we may expect to find in addition to H and K the infra-red doublet with its satellite. Of particular interest are the principal lines produced by the transitions of the type $1\sigma - 1\delta$ and forbidden by the correspondence principle. These transitions, which might be of importance under nebular conditions, should be represented by a doublet in the extreme red, λ 7293.43 and λ 7325.91.

We know that practically all *Ca* atoms in the interstellar cloud are in Ca^{++} form. Of the known Ca^{++} lines only two may have an astrophysical interest: λ 4081.74 and λ 3761.62, as measured in the laboratory by J. A. Anderson[1] and as recently classified by Bowen,[2] but it seems doubtful whether these lines could be detected, on account of their weakness.

V. INTERSTELLAR SODIUM

Sodium is known to be represented in the spectrum of interstellar matter. The ionization potentials of *Na* and Na^+ are 5.1 and 30–35 volts, respectively. We shall adopt for the ionization of *Na* the same T and W as we did for *Ca;* for the reaction $Na^+ \rightarrow Na^{++}$, $T = 16{,}000°$, $W = 1.5 \times 10^{-17}$, and $T_0 = 15{,}300°$ (the same as for Ca^+). In accordance with our previous result, $N_e = 1 \times 10^{-3}$, we obtain $x_0 = 2 \times 10^{-7}$; $x_1 = 1$; $x_2 = 6 \times 10^{-3}$. According to F. W. Clarke and H. S. Washington, the partial abundance of *Na* by atoms for the earth is 1.3 times as large as for *Ca;* therefore if the conditions in interstellar space are the same, *Na* atoms in cosmic clouds are 2×10^{-3} times less abundant than those of Ca^+ ($x_1 = 7 \times 10^{-5}$). Returning to section II, we conclude that if the width of the detached D_1 and D_2 lines is the same as that of the detached K line, the residual intensity of each of the *Na* lines expressed in magnitudes will be one hundred times smaller than that of K, provided the ratio of weights, p_3/p_1, and Einstein's transition coefficients are the same. According to Tolman,[3] the coefficients A_{21} for the D_1 and D_2 lines are 6×10^6 and 3×10^6; i.e., they are at least ten times smaller than for the K line. The ratio of the weights is the same as for K. This diminishes the residual intensity for D_1 and D_2 to 1/500 of its value for K; in other words,

[1] *Astrophysical Journal*, **59**, 76, 1924.

[2] *Loc. cit.* [3] *Physical Review*, **23**, 702, 1924.

if all these calculations are correct, detached *Na* lines could not be observed at all. On the other hand, we know that the observed intensities of the detached lines are not much smaller than those for K. No reasonable change in our assumptions can solve this difficulty; even if we suppose that $\Delta\lambda$ for the D lines is five times smaller than that for K and that the relative abundance of *Na* is five times larger than that of *Ca*—a supposition that seems almost too radical—the intensity of the D lines for the most distant stars will be not more than $0^{m}.1$.

Observational evidence concerning the detached sodium lines is very meager. Miss Heger[1] has shown that in all spectroscopic binaries examined by her stationary Ca^{+} lines are accompanied by stationary *Na* lines. On the other hand, binaries, like π^{4} Orionis (type B3), having oscillating Ca^{+} lines, have also oscillating *Na* lines.

Twenty stars of different spectral types were observed at the Yerkes Observatory in the region of the D lines. The average intensities expressed on a scale similar to that used for the Ca^{+} lines are given below:

Type.....	B1	B2	B3	B5	A0	A2	G	K
No.....	2	3	2	3	5	2	2	1
Aver. int.....	2	5	2	2	4.5	7	10	20

While the increase in intensity f r the later types is not nearly as rapid as for H and K, the general character of the intensities is the same; the "stellar" lines gradually merge into those of interstellar origin, at or near subdivision B3.

Utilizing Miss Heger's descriptions of the D lines in various stars, as well as those of J. S. Plaskett[2] and of P. W. Merrill,[3] we have, together with the Yerkes observations, twenty rough estimates of the intensity of the D_1 lines in stars of types earlier than B5. Of these, 7 are Yerkes observations, 7 are from Victoria, 6 from Lick Observatory, and 1 from Mount Wilson. Dividing these into two groups according to the strength of D_1, we find:

Weak D lines: No. of stars, 11; aver. int. of K = 2.5
Strong D lines: No. of stars, 9; aver. int. of K = 5.2

[1] *Lick Observatory Bulletin*, **10**, 59, 1919; 141, 1921.

[2] *Publications of the Dominion Astrophysical Observatory, Victoria*, **2**, 340, 1924.

[3] *Annual Report of the Director of the Mount Wilson Observatory*, p. 124, 1926–1927.

The strong correlation between the intensities of D_1 and K is obvious. However, this may not apply to every individual star. J. S. Plaskett remarks[1] that the strength of the lines in early spectral subdivisions shows a "much greater variation in character than occurs in the calcium lines."

The single observation from Mount Wilson[2] draws attention to the fact that "the sodium lines are remarkably strong in the Wolf-Rayet stars H.D. 193793, of type Oa." The observed intensity of K is 7.2—the highest values recorded for any Oa star, and among the highest values recorded for any star having detached lines.

As far as the observations go, it seems certain that the detached *Na* lines have the same origin as the detached Ca^+ lines, and that their intensities are of the same order for stars situated at equal distances from the Sun.

The theoretical difficulty in accounting for the observed strength of the detached *Na* lines makes it necessary to revise our postulates.

It is doubtful that it would be possible to obtain an increase of x_0 for *Na* and a decrease of x_1 for *Ca* by changing the theoretical temperature of interstellar radiation. An attempt to vary T has given no useful results: on the contrary, the ratio $x_0(Na)/x_1(Ca)$ seems to be very stable.

The difficulty might be satisfactorily solved by changing the adopted probability coefficient for the D lines. We have not yet any trustworthy determination of these coefficients for sodium; the laboratory data (Bevan-Ladenburg, Harrison) deal with the relative values of A_{ji} and theoretical investigations involve arbitrary assumptions in applying the correspondence principle. On account of that, the lack of success in the interpretation of detached sodium lines cannot be very serious—we have to wait for new physical data.

VI. THE EFFECT OF NEARBY STARS

Other causes contributing to the ionization of interstellar gas are cosmic rays and stimulating radiation of individual nearby stars. The density of cosmic radiation as deduced from Millikan's data is approximately 4×10^{-15} erg/cm^3, i.e., only twenty times less than that of interstellar radiation as derived above from Van Rhijn's

[1] *Loc. cit.* [2] *Loc. cit.*

data. But it should be borne in mind that the ionizing efficiency of "cosmic rays" is relatively small, owing to their extreme hardness. It has been shown elsewhere that the ionization of gas having a density of 10^{-24} and being in statistical equilibrium with the radiation will be negligible;[1] for hydrogen $x = 10^{-3}$.

The ionization of interstellar gas in the vicinity of individual stars will certainly differ somewhat from the mean ionization throughout the cloud. If an early B star intervenes, the temperature of the ionizing radiation remains the same; only the dilution factor is increased by the nearness of the exciting star. The number of Ca^{++} atoms increases at the expense of Ca^{+}; thus the absorption coefficient for the cosmic K line decreases. The intervening star will therefore diminish the intensity of the detached K line; but having in view the enormous distance traversed by stellar rays in the cosmic cloud, we can reasonably assume that the relative effect of the radiation of individual stars on a cosmic line is quite negligible. This reasoning is valid only if the density of interstellar matter does not increase in the vicinity of a star. If, owing to the combined effect of gravitation and radiation pressure of a star, the density of interstellar gas increases, the star in question may affect the intensity of the detached K line in its own spectrum quite appreciably. Suppose the dilution factor W for such a condensation increases by a factor 10^{p} and the total gaseous density by the factor 10^{q}. Then x_1 increases 10^{-p+q} times. If we assume that the condensation as projected on the line of sight has a thickness of 0.1 parsec (this being the average thickness of the nebular shells of planetaries), then from sections I and II the supplementary absorption of the K line as produced by a condensation will be $3.4 \times 10^{-p+2q-5}$ in stellar magnitudes. Suppose further the factor W to be of the same order as that of the planetary nebulae (mean distance from the nucleus, 0.1 parsec),[2] i.e., $W = 4 \times 10^{-11}$. To obtain under such circumstances ($p = 6$) an absorption $\Delta m = 0^{m}.5$, q must be equal to 5. The total density of the condensation then turns out to

[1] Gerasimovič, *Proceedings of the American Academy of Arts and Sciences*, **62**, 182, 1927; *Harvard Reprint*, No. 39.

[2] Gerasimovič, *Proceedings of the American Academy of Arts and Sciences*, **62**, 166, 1927; *Harvard Reprint*, No. 38.

be 10^{-22}, which is to be compared with the average density of planetaries ($\rho = 10^{-20}$ to 10^{-19}). We see that nothing precludes the possibility of separate clouds enveloping the individual stars and giving an absorption effect of the right order, too rarefied, however, to give a nebular emission effect. Such condensations may have been formed by the expulsion of Ca^+ atoms from the hottest stars, as suggested by Milne and others.

VII. OBSERVATIONAL EVIDENCE

Eddington has suggested[1] a method for testing whether there are any such "smoke screens" of calcium near the stars.

Suppose for a moment that a calcium screen is formed. Will it move with the star? Observations show that this is not the case. Otherwise the radial velocities from the detached lines would be approximately the same as those of the stars. Observations disprove this. The O stars as well as the earliest B's move quite independently of the calcium masses, while a slight correlation in subdivision B3 is readily explained by the effect of blending of stellar and detached lines.

If, on the other hand, the calcium atoms emitted by a moving star are caught by the interstellar cloud and brought to relative rest, the smoke screen will trail behind. In such a case it is obvious that receding stars would exhibit much stronger calcium lines than approaching ones.

To test this effect we have computed the residual radial velocities for 135 stars of type O to B2 and for 120 stars of type B3. The data of observation were chiefly taken from the Yerkes catalogue of radial velocities[2] and were supplemented by published results from Victoria, Mount Wilson, and Lick observatories. Table IV shows the average intensities for groups of stars arranged by their residual radial velocities. The component of the solar motion was computed from $A = 270°$; $D = +30°$; $V_\odot = 20$ km/sec. No K-term was applied to the resulting velocities.

It is certain that there exists no "smoke-screen effect." Stars which approach us do not have fainter lines than those which recede.

[1] *Proceedings of the Royal Society of London*, A, 111, 445, 1926.

[2] *Astrophysical Journal*, 64, 1, 1926.

The first part of the table reveals an interesting increase in I for the fastest-moving stars, irrespective of the direction of that motion. This is doubtless due to the fact, noticed first by J. S. Plaskett, that the faintest B stars move, on the average, much faster than the brighter ones. These faint B's are also the most distant ones, and the observed increase in I is therefore simply a consequence of the

TABLE IV

AVERAGE INTENSITIES OF DETACHED K LINE ARRANGED ACCORDING TO THE RESIDUAL RADIAL VELOCITIES OF THE STARS

Limits of Residual Radial Velocity	Mean Residual Radial Velocity	Number	Average Intensity
Type O–B2			
−20 to −∞	−41.6	20	4.3
−10 −20	−14.4	9	3.8
− 5 −10	− 7.4	10	2.6
0 − 5	− 2.2	16	3.2
0 + 5	+ 1.8	17	3.1
+ 5 +10	+ 7.3	11	2.9
+10 +20	+14.4	30	3.4
+20 to +∞	+30.0	22	3.7
Type B3			
−20 to −∞	−31.0	5	2.8
−10 −20	−12.7	10	3.2
− 5 −10	− 7.6	5	3.6
0 − 5	− 2.5	17	3.3
0 + 5	+ 2.4	29	2.6
+ 5 +10	+ 7.1	21	2.5
+10 +20	+13.1	20	2.3
+20 to +∞	+34.2	13	1.7

well-known distance effect. For the B3 stars this effect is not noticeable.

Observational evidence does not favor the existence of any condensations around the stars, and we are probably correct in attributing all absorption to interstellar space.

VIII. THE SUBSTRATUM

The average distance of B stars of the tenth magnitude is at least 1000 parsecs; they are accordingly near the border of our

local system. The great strength of the detached K line in the spectra of O stars and especially in the P Cygni stars, as compared with B stars of the same apparent magnitude, seems to indicate that the *Ca* cloud extends far away toward the general galactic field. On the other hand, the lack of any decrease in β for faint stars proves that, with due allowance for local irregularities, the density of the *Ca* cloud is uniform if we neglect a possible change in T and W for very distant parts of the local system.[1] All evidence seems to show that we are dealing with a gaseous substratum embodying our galaxy and perhaps even extending far beyond it—a "continuum" rather than a cloud.

This hypothesis very satisfactorily accounts for all observational data; it need not necessarily be concluded that interstellar matter is segregated into separate, dynamically independent clouds or condensations. Measurements of radial velocity have established that the calcium masses are outside the stellar atmospheres, and that the motions of star and calcium are independent. Stars near one another in the sky have a tendency to exhibit equally strong detached lines, and the Ca^+ radial velocities display the same tendency. This could be explained by the hypothesis of separate clouds. Now that it is known that absorption can and must occur anywhere in space, the idea of separate clouds loses its strength. Similar intensities for nearby stars probably are a result of the well-known tendency of B-type stars to form isolated clusters in space. This very satisfactorily explains such anomalous intensities as the weak lines predominant in Orion and the strong ones in Cygnus and Cepheus. The similarity of the residual radial velocities was not so easy to explain. However, after B. Lindblad's and J. H. Oort's discovery of a rotational term in the radial velocities of different objects, the whole question took on a very different light. This matter will be discussed in the next section. It will be shown there that most, or perhaps all, of the residual radial velocities from the Ca^+ lines vanish if they are corrected for the effect of galactic rotation.

The interstellar substratum appears to be of rather limited dynamical importance. Its density per parsec is only 0.00015 $\odot$

[1] It can be shown that if the total density decreases as r^{-n} the observed β decreases as r^{-2n}.

(taking $\rho = 10^{-26}$), as compared with 0.045 stars for the general stellar density derived by Kapteyn for the nearest parts of our system near the galactic plane.[1] Even allowing for possible errors in the assumed relative abundance of *Ca*, the total mass of our interstellar substratum cannot be higher than 1/100 of the total stellar mass per unit volume. The gravitational effect of interstellar matter is therefore small; the effect of its resistance to the motion of stars in their cosmic orbits is negligible.

If the interstellar matter is really a continuum extending uniformly in all directions, it could provide a suitable source for the hard cosmic radiation, which, according to Millikan's repeated observations, does not show any marked daily variation. "Cosmic rays," if originated by the system of isolated stars or of nebular condensations, would certainly show a pronounced daily variation depending upon the variable position of the Milky Way over the horizon. These rays, owing to their extreme hardness, cannot be absorbed appreciably by the atoms, but can be only scattered by free electrons. J. J. Thomson's scattering coefficient for free electrons is $\sigma = 6.1 \times 10^{-25} N_e$, where N_e is the number of free electrons for the interstellar medium per cm^3; therefore, $\sigma = 6.1 \times 10^{-28}$. A. H. Compton's coefficient of true absorption due to this scattering is $S = \sigma \times a/(1+2a)^2$, where $a = 0.0242/\lambda$ and λ is expressed in angstroms. For $\lambda = 4 \times 10^{-4}$, which corresponds to the lower end of the cosmic-ray spectrum, and for the interstellar medium, $S \sim 10^{-30}$. This gives an amount of extinction that is entirely negligible even for a distance of 50,000 parsecs. We can therefore consider interstellar gas as transparent for cosmic rays. Let us suppose that interstellar gas producing cosmic rays is confined to the galactic system, having a radius of 50,000 parsecs. The density of penetrating radiation is 4×10^{-15} erg/cm^3; calculating according to our hypothesis, the energy of the penetrating radiation as produced by transparent interstellar gas is found to be 8×10^{-75} erg/sec. If, as it is natural to suppose, following Millikan, the penetrating radiation is produced by transmutation of *H* into *He*, when a quantum 5×10^{-5} erg is released, the total number of transmutations per cm^3 and second will be of the order of 10^{-70}. If interstellar matter of such a density as is

[1] *Astrophysical Journal*, **55**, 302, 1922.

determined above were composed of hydrogen only, the total number of transmutations per atom and second would be 10^{-66}; each atom, therefore, will undergo transmutation only once in 10^{59} years.

Except for monochromatic absorption and for absorption in short waves beyond the series limits, the interstellar gas will be transparent. The dimming of stellar light due to the scattering by free electrons in the visual part is quite insignificant. Even for the distance of the Andromeda nebula (300,000 parsecs) the loss of light is only $0^m.0005$. This result is in harmony with Shapley's observations on globular clusters, which disproved the existence of any appreciable general absorption in space.

IX. GALACTIC ROTATION

J. H. Oort[1] has demonstrated that all distant stars show in their residual radial velocities a term of the form $rA \sin 2(l-325°)$ and

TABLE V

ROTATIONAL EFFECT IN Ca^+ VELOCITIES

Limits of Galactic Longitude	Mean Galactic Longitude	Number	Mean Residual Radial Velocity	Computed Radial Velocity	Remarks
319°–342°	329°	4	+ 4.2	+0.1	
0 – 24	12	2	+ 5.0	+4.7	
35 – 49	42	15	+ 1.3	+1.7	
53 – 69	64	17	− 2.3	−2.2	
70 – 78	72	11	+ 2.6	−3.5	
83 – 95	88	8	− 8.1	−5.5	
103 –117	110	6	−10.0	−5.6	
127 –158	143	10	+ 0.8	−1.0	
162 –168	165	7	+ 1.9	+2.8	
170 –204	176	23	+ 0.7	+4.1	Orion region

that this term can be interpreted as being due to rotation of all stars around a distant central mass in galactic longitude 325°.

Oort computed also the value of rA from 40 Ca^+ velocities published by Plaskett. These tended to show that the "calcium clouds" participate in the general rotation with a value of $rA = 5.6 \pm 1.5$ km/sec. (p.e.).

[1] *Bulletins of the Astronomical Institutes of the Netherlands*, **3**, 275, 1927.

J. S. Plaskett[1] has recently confirmed Oort's deductions for the B- and O-type stars.

We have carried through a similar computation for all available Ca^+ velocities derived from stars of type O–B2. The B3 stars are somewhat affected by the stellar line, and we have not used them in our computations.

The total number of Ca^+ velocities used was 103. This included Yerkes and Victoria data for single stars as well as mean velocities from "stationary" lines for a number of spectroscopic binaries. The component of the solar motion, computed from $A = 270°$, $D = +30°$, $V_\odot = 20$ km/sec., was subtracted from each velocity. The stars were then grouped according to galactic longitude, and means were taken as shown in Table V and in Figure 2.

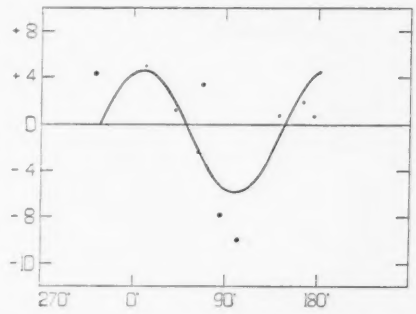

FIG. 2.—The effect of galactic rotation in the residual radial velocities from detached calcium lines. The abscissae are galactic longitudes and the ordinates give the corresponding mean radial velocities.

We then computed for each of the ten groups an equation of the form

$$K + rA \sin 2(l - 325°) = \bar{\rho} .$$

Solving these equations by the method of least squares, we obtain

$$rA = +5.3 \pm 1.0 \ (p.e.) ,$$
$$K = -0.6 \pm 0.7 \ (p.e.) .$$

The existence of a rotational effect is beyond doubt. The negative value of K is of course quite illusory, in view of the fact that the probable error is ± 0.7 km/sec.

The stars from which the Ca^+ velocities were taken, fall into the following groups:

46 stars of type Bo–B2, brighter than 6.21; $rA = 5.9$
25 stars of type Bo–B2, fainter than 6.20; $rA = 16.7$
32 stars of type O , $rA = 17.1$
Total 103, weighted mean ..... $rA = 12.0$

[1] *Monthly Notices of the Royal Astronomical Society*, **88**, 395, 1928.

The value of rA for the various groups was taken from Plaskett's data. The weighted mean value of rA for all 103 stars is $+12.0$ km/sec.

This is almost exactly twice the value derived from the Ca^+ velocities. The difference in rA may be due to a difference in r or in A, or in both.

A priori we have no right to assume that A has the same value for stars as for separate atoms. Indeed, the motions of the stars are regulated by gravity only, while the atoms are affected by radiation pressure as well.

However, we know that there is no very bright portion in the Milky Way that corresponds with the hypothetical central condensation. Moreover, Seares believes that the central condensation is obscured by dark nebulae. If this is true there will be no appreciable gradient of radiation pressure acting upon the atoms of our substratum, and A may be assumed to be the same as for the stars. The effective distance of the Ca cloud is therefore one-half of the average distance of the stars—exactly what we might have expected from the constancy of the coefficient of absorption all over space.

The residual radial velocities from the Ca^+ lines, after correction for the effect of rotation, are in most cases very small. The remaining differences are doubtless partly due to individual differences in r (this is clearly shown by the small value of $rA \sin 2[l-325°]$, in Orion, where the stars are comparatively near to the Sun).

X. CONCLUSIONS

Analysis of the physical properties of interstellar calcium leads us to believe that the whole galactic system is immersed in a gaseous substratum consisting of atoms of various elements. The density of this substratum is of the order of 10^{-26}. The separate atoms, while obeying the ordinary gas laws, participate in a rotational movement around a distant central mass in galactic longitude 325°. The observed rotational acceleration seems to be the same as for the stars, so that the atoms are probably not subjected to any appreciable special radiation pressure from the central mass.

The idea of a uniform gaseous medium in interstellar space is in no way unusual or unexpected. In fact, it would be much more

difficult to imagine space entirely devoid of matter. Indeed, we know that the Sun is constantly throwing out atoms of calcium and of other elements in form of prominences. The absence of certain light gases in the atmospheres of the planets and the absence of any atmospheres on the satellites and asteroids of our solar system have always been explained by the escape of fast-moving molecules from under the gravitational attraction of the smaller bodies. Such atoms and molecules must fill space, and it is by no means necessary or even probable that such diffuse matter is limited to the solar system. We have direct observational evidence that matter in comets' tails is driven away from the Sun under the force of radiation pressure. Milne has shown theoretically that similar processes will take place in the case of other elements, the atoms being thrown away from the Sun and consequently also from many stars. Finally, it may be noted that a medium like the one described in this article has formed the essence of cosmological hypotheses, like the one of MacMillan.[1]

The physical properties of this medium were derived entirely from observational data. The fact that this medium turns out to be sufficiently transparent to explain Shapley's results on globular clusters, that it shows the effect of galactic rotation of exactly the right order of magnitude, and that it reconciles all observations on detached spectral lines strongly commends the proposed hypothesis.

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May 1928

[1] *Astrophysical Journal*, **48**, 35, 1918.

THE ABSORPTION SPECTRUM OF CS_2 IN THE NEAR ULTRA-VIOLET

By EARL D. WILSON

ABSTRACT

The absorption spectrum of carbon disulphide vapor in the near ultra-violet has been photographed under varied experimental conditions in such a manner as to exhibit a rather complete development of the band system *over the range of wave-lengths* λ 2900–λ 3800.

The wave-lengths of about 650 *absorption lines* have been *measured*, and their *relative intensities estimated from photometric curves.*

Certain regularities of structure in the band system have been pointed out, and some *series with constant differences in frequency recorded.*

INTRODUCTION

For more than a quarter of a century investigators have been baffled by the intricacy of the molecular spectra of the familiar compound, carbon disulphide. Only meager data have been published up to the present time, so that it is necessary for each new investigator to begin all the work afresh.

The work of W. W. Coblentz,[1] which compares the infra-red emission spectrum of burning CS_2 with the absorption spectrum of the liquid phase, constitutes about all of the data available in the region of long wave-lengths. Covering the spectral region from 2 μ to 15 μ, he records only the three absorption bands at 4.6 μ, 6.75 μ, and 11.65 μ, although his own curve indicates the presence also of other bands at 3.2 μ, 6.3 μ, and 13.4 μ.

More than fifty years ago Liveing and Dewar[2] investigated in a qualitative manner the absorption spectra of a number of gases and vapors. They found bands for CS_2 extending over the region from λ 3360 to λ 3020. About fifteen years later J. Pauer[3] measured the wave-lengths of twenty-four members of this system. In 1919 Miss Pauly[4] with improved apparatus succeeded in measuring as many as eighty-five lines ranging from λ 3589 to λ 2919.

Aside from measurements of absorption coefficients of CS_2 liquid

[1] *Physical Review*, **24**, 72, 1907.

[2] *Proceedings of the Royal Society*, **35**, 71, 1883.

[3] *Annalen der Physik*, **61**, 363, 1897.

[4] *Astrophysical Journal*, **50**, 155, 1919.

for ultra-violet light and the additional fact[1] that a second strong absorption band occurs with maximum at about λ 2200, the foregoing summarizes our present state of published knowledge concerning the molecular spectra of CS_2.

The writer has therefore thought it worth while to investigate the near ultra-violet absorption spectrum of CS_2 vapor with the aid of modern equipment in order that data of wave-lengths and intensities might be prepared more adequate to ultimate quantum analysis.

EXPERIMENTAL PROCEDURE

The carbon disulphide used in this investigation was chemically pure, being colorless and having no unpleasant odor. The vapor was confined in a glass tube with end plates of fused quartz secured in position by an acetic-acid-gelatine cement. Liquid, introduced through a side tube, occupied a bulb attached to the opposite wall of the vapor chamber. The bulb was provided with a long stem to facilitate its immersion in a bath of desired temperature. Other side tubes made possible the evacuation of the system or the circulation of vapor. Five such vapor chambers, ranging in length from 2 to 60 cm, provided convenient means of varying the length of the column of vapor.

The under-water spark was employed as a source of continuous radiation in the ultra-violet. For preliminary determination of the conditions most suitable for developing the band system *in extenso*, a Fery spectrograph of small dispersion (roughly 40 A per mm in ultra-violet) was found quite expedient. A series of photographs of the absorption spectrum was made, varying successively the different disposable parameters of the equipment. The length of vapor column was varied by substituting different vapor chambers. The vapor pressure was altered either by use of a vacuum pump or by changing the temperature of the bath in which the bulb was immersed. It so happens that the freezing point of CS_2 is extremely low, $-110°$ C., and the liquid could be readily frozen by its own evaporation at low pressure. The vapor pressure of the ice was so low that no absorption occurred. Other temperatures were secured by salt and ice baths and by baths of water warmed stepwise from

[1] Bruhat and Pautheimer, *Comptes rendus*, **179**, 154, 1924.

0° to 100° C. At these higher temperatures, the chamber was wrapped in asbestos paper and warmed with Bunsen flames so that no condensation of CS_2 vapor could take place on the end plates. Pressures as high as 3 or 4 atm. were obtained in this manner, as estimated by consulting Regnault's tables for the vapor pressure of CS_2 at various temperatures.

Such experiments showed: first, that the structure of the band system did not depend critically either upon the temperature in itself or upon the pressure of air or upon the length of exposure; second, that the intensity of the system was a consistent function of the number of molecules traversed by the incident radiation, by whatever means this number was varied.

The conditions suitable for the absorption of the bands having been determined, the significant exposures were repeated with a Hilger E-1 spectrograph. The arc spectrum of iron was used as a standard of wave-length.

By means of a Moll microphotometer, intensity-curves both of the CS_2-absorption lines and iron-emission lines were traced successively upon the same print. Altogether, eight final prints were completed which faithfully portrayed the structure of the system in overlapping portions from λ 3800 to λ 2900. Figure 1 is a copy (with iron lines omitted) of one of the prints for the central, most intense portion of the band system. Figure 2 represents the least refrangible portion of the whole band system.

The wave-lengths of about six hundred and fifty absorption lines were measured partly from these photometer prints and partly from the original plates. The results are recorded in Table I. The error to be expected is of the order of 0.05 A, as judged from overlapping sections from separate plates.

The relative intensities of the lines were estimated linearly from the corresponding amplitudes on the photometric curves. The weakest lines are counted unity; and the line of maximum intensity, 1000 units.

Discussion of structure of the band system.—In general, the intensity of the system is greatest in the central portion, decreasing with some uniformity toward both longer and shorter wave-lengths. By far the most intense bands are included in the range between

about λ 3100 and λ 3300. In this region there are numerous maxima of intensity having values of the order of 300 to 1000. At λ 3275 there is a sudden decrease from an intensity of about 750 to those of value about 150, and after the group of lines at λ 3320 the intensities decrease rapidly from about 30 or 40 to unity. On the more refrangible side, the decrease of intensity proceeds without any unusual discontinuities.

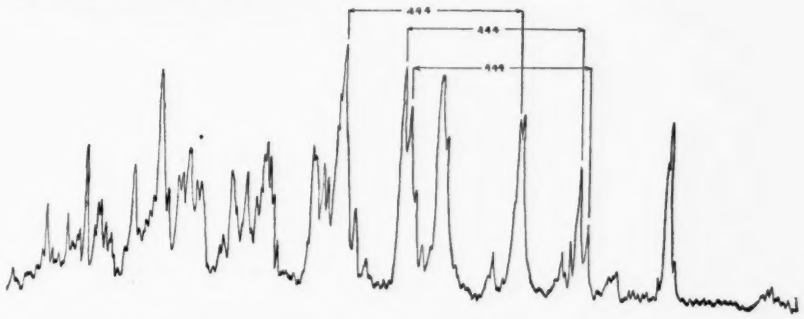

FIG. 1.—Photometer-curve of the central part of the CS_2 band system

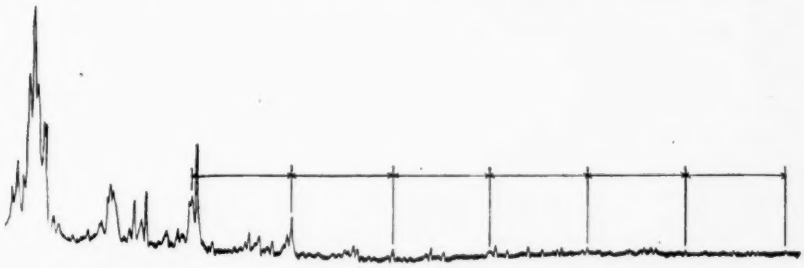

FIG. 2.—Photometer-curve of the long-wave part of CS_2 band system

It is a very noticeable fact that the more refrangible end of the system is much less sharply defined than the end of longer wave-lengths. Also no standard band structure is evident, but the appearance is more suggestive of a mixture of resolved and unresolved atomic lines.

Certainly there are very few obvious regularities in frequency-differences in the CS_2 absorption spectrum. Perhaps the series that appear most promising under low dispersion are those extending to longer wave-lengths from the neighborhood of λ 3275. Under higher dispersion, the members of the series are composed each of a number

TABLE I

Absorption Bands of CS_2 in Near Ultra-Violet

λ(I.A.)	I	ν(Vac.)	λ(I.A.)	I	ν(Vac.)	λ(I.A.)	I	ν(Vac.)
2907.30	2	34386.1	2960.16	5	33772.1	3022.78	100	33072.5
2908.10	3	34376.7	2960.42	5	33769.2	3023.99	100	33059.2
2908.57	4	34371.1	2961.53	1	33756.5	3024.97	80	33048.6
2909.37	5	34361.7	2962.16	1	33749.3	3025.73	36	33039.3
2909.83	5	34356.2	2963.77	8	33731.0	3026.99	36	33026.4
2910.40	5	34349.5	2964.63	10	33721.2	3027.32	43	33022.9
2911.58	8	34335.6	2965.02	20	33716.8	3027.77	55	33018.0
2912.40	9	34325.9	2965.78	16	33708.1	3028.24	70	33012.9
2913.41	9	34314.0	2966.35	23	33701.7	3028.85	33	33006.3
2913.88	6	34308.5	2967.66	23	33686.8	3029.63	33	32997.7
2914.39	5	34302.5	2968.02	20	33682.7	3030.11	40	32992.4
2915.33	4	34291.4	2969.02	13	33671.3	3031.74	65	32974.8
2915.79	5	34286.0	2969.32	10	33667.9	3032.28	75	32968.9
2917.24	4	34269.0	2970.55	23	33654.0	3033.33	120	32957.5
2917.59	4	34264.9	2971.05	33	33648.3	3034.27	120	32947.3
2918.18	3	34257.9	2972.98	23	33626.5	3036.40	120	32924.2
2919.36	4	34244.1	2973.82	26	33617.0	3037.29	75	32914.5
2920.36	4	34232.4	2974.48	20	33609.5	3038.22	110	32904.4
2920.90	4	34226.0	2975.14	10	33602.1	3038.70	90	32899.3
2921.51	3	34218.9	2976.15	7	33590.7	3039.68	80	32888.6
2922.48	5	34207.5	2977.28	20	33577.9	3041.08	70	32873.5
2922.84	5	34203.3	2977.69	23	33573.3	3042.25	33	32860.9
2923.97	8	34190.1	2978.55	26	33563.6	3043.88	23	32843.3
2924.33	10	34185.9	2979.33	40	33554.8	3044.80	36	32833.4
2924.93	8	34178.9	2980.6	10	33541	3045.46	30	32826.2
2925.67	8	34170.2	2982.40	7	33520.3	3046.88	23	32810.9
2926.19	5	34164.2	2982.98	10	33513.8	3048.90	50	32789.2
2927.15	2	34153.0	2983.23	12	33511.0	3049.72	50	32780.4
2928.16	4	34141.2	2983.53	13	33507.6	3050.76	80	32769.2
2928.60	4	34136.0	2984.50	22	33496.7	3051.74	80	32758.7
2929.11	4	34130.1	2985.26	30	33488.2	3052.33	75	32752.4
2929.82	4	34121.8	2985.70	23	33483.2	3053.23	55	32742.7
2930.81	6	34110.3	2986.74	10	33471.6	3054.41	40	32730.0
2931.40	6	34103.4	2987.72	4	33460.6	3055.33	50	32720.2
2931.93	6	34098.4	2989.07	10	33445.5	3056.32	80	32709.6
2932.92	8	34086.9	2990.32	16	33431.5	3056.83	100	32704.1
2933.60	9	34077.9	2991.48	43	33418.5	3057.87	43	32693.0
2933.95	10	34073.8	2992.77	55	33404.1	3058.71	20	32684.0
2934.80	8	34063.9	2993.44	65	33396.7	3059.70	10	32673.5
2935.27	7	34058.5	2994.05	60	33389.9	3060.75	13	32662.3
2935.95	5	34050.6	2995.15	16	33377.6	3061.14	10	32658.1
2937.32	7	34034.7	2996.36	7	33364.1	3062.18	7	32647.0
2938.19	8	34024.6	2997.50	13	33351.4	3063.81	23	32629.6
2938.92	5	34016.2	2998.36	16	33341.9	3064.32	36	32624.2
2939.62	2	34008.1	2999.60	10	33328.1	3065.02	23	32616.7
2941.13	2	33990.6	3000.46	4	33318.6	3065.83	27	32608.1
2941.96	1	33981.0	3001.15	4	33310.9	3067.34	43	32592.1
2942.23	1	33977.9	3001.70	7	33304.8	3068.25	75	32582.4
2942.80	2	33971.3	3002.97	10	33290.7	3068.63	75	32578.4
2943.17	3	33967.1	3003.23	10	33287.8	3069.05	80	32574.1
2944.20	3	33955.2	3004.42	7	33274.6	3070.82	120	32555.1
2945.19	8	33943.8	3005.19	5	33266.1	3072.23	185	32540.2
2945.83	10	33936.4	3006.13	13	33255.7	3073.14	160	32530.6
2946.14	10	33932.8	3006.90	13	33247.2	3074.13	90	32520.1
2946.69	8	33926.5	3007.77	23	33237.6	3074.99	100	32510.9
2947.60	6	33916.0	3008.46	23	33229.9	3077.01	145	32489.7
2948.60	9	33904.5	3008.65	43	33227.8	3078.17	145	32477.4
2949.52	6	33893.9	3009.44	60	33219.1	3079.39	120	32464.6
2950.40	10	33883.8	3010.60	13	33206.3	3080.14	240	32456.7
2951.08	7	33876.1	3011.67	20	33194.5	3080.56	360	32451.2
2952.00	16	33865.5	3012.01	20	33190.8	3080.99	290	32447.6
2952.90	10	33855.1	3013.09	43	33178.9	3082.18	225	32435.2
2953.96	7	33843.0	3014.31	27	33165.4	3083.32	135	32423.2
2954.50	8	33836.8	3015.08	50	33157.0	3084.34	135	32412.5
2955.35	7	33827.1	3017.00	43	33135.9	3085.07	170	32404.8
2956.07	5	33818.8	3018.35	13	33121.1	3085.86	105	32396.5
2956.68	4	33811.9	3019.38	33	33109.8	3087.30	305	32381.4
2958.20	6	33794.5	3019.95	36	33103.5	3088.23	120	32371.6
2958.82	8	33787.4	3020.73	46	33094.9	3089.85	65	32354.7
2959.33	10	33781.6	3022.10	65	33080.0	3091.66	305	32335.7

TABLE I—*Continued*

λ(I.A.)	I	ν(Vac.)	λ(I.A.)	I	ν(Vac.)	λ(I.A.)	I	ν(Vac.)
3092.49.....	475	32326.9	3153.96.....	410	31697.0	3231.82.....	265	30933.4
3092.99.....	475	32321.7	3154.42.....	395	31692.4	3233.4.....	410	30918
3093.51.....	290	32316.5	3155.7.....	36	31680	3234.50.....	760	30907.8
3094.67.....	90	32304.3	3157.07.....	65	31665.8	3235.28.....	760	30900.3
3095.88.....	170	32291.6	3157.58.....	65	31660.7	3236.84.....	130	30885.4
3096.82.....	160	32281.8	3158.59.....	110	31650.6	3237.90.....	55	30875.3
3097.78.....	145	32271.8	3159.4.....	175	31642	3238.64.....	26	30868.3
3099.11.....	160	32258.0	3160.1.....	145	31635	3239.80.....	26	30857.2
3099.79.....	290	32250.8	3161.05.....	330	31625.9	3240.68.....	13	30848.8
3100.39.....	360	32244.6	3161.50.....	365	31621.4	3242.34.....	26	30833.0
3100.83.....	280	32240.1	3161.94.....	425	31617.0	3243.5.....	40	30822
3102.41.....	185	32223.7	3162.90.....	290	31607.0	3244.4.....	80	30813
3103.28.....	145	32214.6	3163.97.....	210	31596.7	3245.70.....	185	30801.1
3104.75.....	27	32199.4	3165.24.....	450	31584.1	3246.65.....	105	30792.1
3105.43.....	27	32192.3	3166.56.....	200	31570.9	3247.88.....	325	30780.5
3105.98.....	27	32186.6	3167.71.....	330	31559.4	3249.61.....	280	30764.1
3106.64.....	13	32179.8	3168.80.....	370	31548.6	3250.18.....	320	30758.4
3107.37.....	27	32172.2	3169.67.....	500	31539.9	3250.56.....	460	30755.1
3109.25.....	90	32152.8	3170.25.....	585	31534.1	3252.42.....	305	30737.5
3110.40.....	40	32140.9	3171.08.....	460	31525.9	3255.42.....	40	30709.2
3112.20.....	65	32122.3	3171.98.....	330	31516.9	3255.98.....	65	30703.9
3112.82.....	65	32115.9	3172.89.....	130	31507.9	3257.60.....	80	30688.6
3113.62.....	65	32107.7	3173.83.....	36	31498.6	3258.28.....	105	30682.2
3114.26.....	55	32101.1	3174.4.....	36	31493	3259.50.....	105	30670.7
3114.97.....	90	32093.7	3175.20.....	55	31485.0	3260.42.....	130	30662.1
3115.45.....	90	32088.8	3176.28.....	26	31474.3	3261.93.....	22	30647.9
3116.27.....	175	32080.3	3177.12.....	55	31466.0	3262.72.....	30	30640.5
3117.07.....	345	32072.1	3178.30.....	13	31454.3	3263.75.....	48	30630.8
3117.69.....	175	32065.7	3179.43.....	55	31443.1	3264.79.....	48	30621.0
3118.57.....	175	32056.7	3180.53.....	185	31432.2	3265.50.....	33	30614.4
3119.30.....	530	32049.2	3181.51.....	595	31422.6	3266.12.....	30	30608.6
3120.11.....	145	32040.9	3182.2.....	545	31416	3267.37.....	43	30596.9
3121.02.....	105	32031.6	3184.18.....	500	31396.2	3268.39.....	46	30587.3
3122.15.....	305	32019.9	3185.33.....	435	31384.9	3269.58.....	30	30576.2
3123.12.....	175	32010.0	3185.87.....	300	31379.5	3270.38.....	41	30568.7
3123.43.....	160	32006.8	3186.65.....	495	31371.9	3271.53.....	65	30558.0
3124.43.....	185	31996.5	3187.46.....	665	31363.9	3274.06.....	565	30534.3
3124.96.....	200	31991.2	3188.11.....	790	31357.5	3274.77.....	745	30527.7
3126.61.....	595	31974.3	3189.49.....	1000	31343.9	3275.94.....	145	30516.8
3127.90.....	90	31961.1	3192.30.....	320	31316.4	3276.82.....	65	30508.6
3128.50.....	250	31955.0	3193.75.....	55	31302.1	3278.12.....	55	30496.5
3129.28.....	345	31947.0	3194.40.....	95	31295.8	3279.71.....	26	30481.7
3129.97.....	360	31940.0	3194.85.....	120	31291.3	3280.26.....	50	30476.6
3131.19.....	265	31927.5	3195.68.....	55	31283.2	3281.52.....	40	30464.9
3131.97.....	160	31919.6	3196.37.....	36	31276.5	3283.15.....	22	30449.8
3132.45.....	190	31914.7	3197.70.....	13	31263.5	3283.80.....	26	30433.8
3133.12.....	90	31907.8	3198.39.....	26	31256.8	3285.12.....	33	30431.6
3134.2.....	40	31897	3199.71.....	36	31243.8	3285.70.....	18	30426.2
3135.41.....	145	31884.5	3201.12.....	36	31230.1	3287.82.....	43	30406.5
3135.71.....	500	31881.5	3202.41.....	200	31217.5	3288.69.....	32	30398.5
3136.88.....	90	31869.6	3203.89.....	665	31203.1	3289.42.....	46	30391.8
3137.64.....	505	31861.9	3204.38.....	820	31198.3	3289.82.....	41	30388.1
3139.1.....	200	31847	3206.08.....	665	31181.8	3290.41.....	38	30382.6
3140.0.....	200	31838	3207.75.....	425	31165.5	3291.20.....	48	30375.3
3140.72.....	200	31830.6	3209.32.....	200	31150.3	3292.32.....	40	30365.0
3141.62.....	240	31821.5	3211.0.....	145	31133	3293.41.....	13	30355.0
3142.46.....	305	31813.0	3211.7.....	200	31127	3294.69.....	16	30344.0
3143.99.....	790	31797.4	3212.97.....	490	31114.9	3295.26.....	5	30337.9
3144.51.....	565	31792.3	3213.72.....	680	31107.6	3296.97.....	12	30322.2
3144.90.....	495	31788.3	3214.35.....	695	31101.5	3297.64.....	36	30316.0
3145.46.....	330	31782.7	3215.72.....	425	31088.3	3298.49.....	60	30308.3
3146.09.....	385	31776.3	3218.21.....	120	31064.2	3299.56.....	90	30298.4
3147.00.....	200	31767.1	3218.86.....	80	31057.9	3300.15.....	95	30293.0
3147.46.....	230	31762.5	3220.28.....	55	31044.2	3301.31.....	105	30282.3
3147.94.....	265	31757.6	3221.36.....	26	31033.8	3302.41.....	100	30272.2
3148.44.....	255	31752.6	3223.20.....	13	31016.1	3303.74.....	85	30260.0
3149.48.....	365	31742.1	3224.17.....	13	31006.8	3304.38.....	85	30254.2
3150.37.....	430	31733.1	3225.88.....	85	30990.3	3305.04.....	65	30248.1
3150.94.....	545	31727.4	3226.58.....	95	30983.6	3305.48.....	55	30244.1
3151.20.....	460	31724.8	3227.45.....	200	30975.3	3307.02.....	55	30230.0
3152.06.....	330	31716.1	3230.22.....	65	30948.7	3309.06.....	8	30211.4
3152.80.....	410	31708.7	3231.12.....	165	30940.1	3311.59.....	7	30203.5

TABLE I—*Continued*

λ(I.A.)	I	ν(Vac.)	λ(I.A.)	I	ν(Vac.)	λ(I.A.)	I	ν(Vac.)
3311.93	7	30188.3	3402.83	5	29378.9	3524.14	1	28367.6
3314.18	10	30164.7	3404.10	6	29367.9	3524.73	1	28362.9
3314.69	8	30160.1	3404.81	6	29361.8	3525.73	1	28354.8
3315.45	8	30153.2	3405.70	7	29354.1	3525.90	1	28353.5
3316.96	18	30139.4	3407.69	7	29337.0	3531.54	1	28308.2
3318.37	33	30126.6	3410.18	3	29315.6	3533.51	1	28292.4
3320.46	120	30107.7	3412.22	8	29298.1	3535.23	1	28278.6
3321.58	135	30097.5	3413.56	8	29286.6	3536.26	2	28270.4
3324.96	85	30066.9	3415.34	6	29271.3	3538.05	1	28256.1
3326.59	75	30052.2	3416.14	8	29264.4	3539.6	1	28244
3327.66	23	30042.5	3417.13	7	29256.0	3540.91	1	28233.3
3328.93	13	30031.1	3417.92	12	29249.2	3542.61	1	28219.7
3329.58	18	30025.2	3418.48	12	29244.4	3544.16	1	28207.4
3329.91	16	30022.2	3419.36	15	29236.9	3546.00	1	28192.8
3330.40	15	30017.8	3420.76	7	29224.9	3548.14	1	28175.7
3331.10	15	30011.5	3423.25	6	29203.6	3549.28	1	28166.7
3333.43	10	29990.5	3425.21	3	29186.9	3550.67	3	28155.7
3335.10	8	29975.5	3427.43	2	29168.0	3552.29	1	28142.8
3335.98	10	29967.6	3428.90	4	29155.5	3554.39	1	28126.2
3337.15	20	29957.1	3430.44	5	29142.4	3556.26	1	28111.4
3338.61	13	29944.0	3431.54	4	29133.1	3559.96	1	28082.2
3339.76	18	29933.7	3432.95	2	29121.1	3565.17	1	28041.2
3341.91	23	29914.4	3436.19	13	29093.7	3566.85	1	28028.0
3343.07	26	29904.0	3437.63	22	29081.5	3568.0	1	28019
3344.01	22	29895.6	3439.29	12	29067.5	3569.15	1	28009.9
3345.35	28	29883.7	3442.00	5	29044.6	3570.10	1	28002.5
3346.04	36	29877.5	3444.25	6	29025.6	3572.00	1	27987.6
3347.07	34	29868.3	3446.00	2	29010.9	3573.26	1	27977.7
3349.01	26	29851.0	3447.5	1	28998	3574.81	1	27965.6
3349.98	15	29842.3	3449.81	5	28978.8	3579.30	1	27930.5
3350.87	12	29834.4	3450.84	5	28970.2	3583.73	1	27895.9
3351.52	8	29828.7	3452.93	3	28952.6	3585.62	1	27881.2
3352.40	7	29820.8	3455.52	1	28930.9	3587.32	1	27868.0
3352.98	5	29815.7	3459.46	4	28898.0	3588.32	1	27860.3
3354.01	4	29806.5	3461.17	5	28883.7	3589.85	1	27848.4
3355.35	8	29794.6	3463.22	5	28866.6	3591.95	1	27832.1
3356.50	15	29784.4	3464.92	12	28852.5	3597.53	1	27789.0
3357.42	23	29776.2	3466.52	16	28839.1	3599.14	1	27776.5
3358.71	12	29764.8	3467.46	12	28831.3	3600.46	1	27766.3
3359.82	5	29755.0	3469.20	7	28816.9	3601.58	1	27757.6
3360.2	13	29752	3469.81	7	28811.8	3603.59	1	27742.2
3361.07	5	29743.9	3472.40	2	28790.3	3621.1	1	27608
3361.89	10	29736.8	3474.14	1	28775.9	3622.42	1	27598.0
3362.65	12	29730.0	3475.06	1	28768.3	3626.72	1	27565.3
3363.32	13	29724.0	3478.45	1	28740.2	3634.16	1	27508.9
3364.09	20	29717.2	3480.88	1	28720.2	3635.52	1	27498.6
3366.66	20	29694.5	3482.00	1	28710.9	3639.52	1	27468.4
3367.40	20	29688.0	3482.93	1	28703.2	3645.12	1	27426.1
3369.08	23	29673.2	3483.53	1	28698.3	3646.38	1	27416.7
3369.79	23	29667.0	3484.8	1	28688	3654.17	1	27358.2
3371.42	13	29652.6	3485.5	1	28682	3656.05	1	27344.1
3372.80	5	29640.5	3486.6	1	28673	3659.3	1	27320
3373.97	9	29630.2	3487.57	2	28665.1	3660.2	1	27313
3376.47	8	29608.3	3489.52	3	28649.1	3664.62	1	27280.2
3377.41	6	29600.0	3490.42	4	28641.7	3670.93	1	27233.3
3378.85	5	29587.4	3491.43	4	28633.4	3671.91	1	27226.0
3379.97	1	29577.6	3492.00	3	28628.7	3675.88	1	27196.6
3380.93	2	29569.2	3494.82	1	28605.6	3677.43	1	27185.2
3381.76	5	29561.9	3496.75	1	28589.8	3679.98	1	27166.3
3382.55	5	29555.0	3498.04	3	28579.3	3681.87	1	27152.4
3382.91	5	29551.9	3500.67	2	28557.8	3693.14	1	27069.5
3384.21	4	29540.5	3502.26	4	28544.8	3711.77	1	26933.7
3387.52	7	29511.6	3503.47	1	28535.0	3716.97	1	26896.0
3388.59	20	29502.4	3504.00	1	28530.7	3718.96	1	26881.6
3390.80	20	29483.1	3504.90	1	28523.4	3721.14	1	26865.9
3392.57	10	29467.7	3506.02	1	28514.2	3731.96	1	26788.0
3393.36	10	29460.9	3509.42	1	28486.6	3737.6	1	26747
3394.66	15	29449.6	3512.62	2	28460.7	3751.5	1	26648
3395.18	7	29445.1	3513.69	2	28452.0	3769.8	1	26519
3396.71	7	29431.8	3514.26	1	28447.4	3773.0	1	26497
3397.67	8	29423.5	3516.30	3	28430.9	3783.9	1	26420
3398.45	4	29416.7	3517.15	4	28424.0	3786.11	1	26404.9
3399.46	5	29408.0	3518.94	6	28409.5			
3401.59	5	29389.6	3522.85	1	28378.0			

of lines. Taking one of the strongest lines from each group, we may form two possible short series:

ν	I	$\Delta\nu$	ν	I	$\Delta\nu$
30682.2	105	399.9	30097.5	135	430.5
30282.3	105	398.6	29667.0	23	430.1
29883.7	28	400.6	29236.9	15	397.8
29483.1	20	401.6	28839.1	16	429.6
29081.5	22	.....	28409.5	6	.....
30527.7	745	430.2			

The fourth difference in the second series is entirely irregular, but the remaining values of $\Delta\nu$ are nearly enough constant to have a possible significance.

An examination of Figure 1 reveals the probable recurrence of similar groupings of lines in the intense part of the spectrum. The following frequency-differences are noteworthy:

ν_1	I	ν_2	I	$\nu_1-\nu_2$
31343.9	1000	30900.3	760	443.6
31198.3	820	30755.1	460	443.2
31181.8	665	30737.5	305	444.3
Mean	.....	.....	.....	443.7

Of course other differences of 444 units have been found, but none so convincing in significance as these above. Unfortunately, this interval does not seem to exist with any accuracy even between other pairs of less intense lines in these same groups.

Figure 2 gives an idea of the distribution of lines in the least refrangible part of the spectrum. In this region there are relatively wide intervals between successive lines so that analysis seems possible. Three series have been traced back into shorter wavelengths. They give evidence of reality, as may be judged from the data shown in Table II.

The first series in Table II is rather more satisfactory in its completeness and consistency than the other two. It is to be observed that the first few members of each series have a noticeably larger interval than the remaining members—a phenomenon not unusual in certain band spectra. The means of $\Delta\nu$ given above are computed with omission of these irregular intervals.

An attempt to extend the location of series which would consistently account for the majority of the lines measured has been fruitless. Apparently there are overlapping systems which the present resolution fails to disentangle. The proximity of successive lines necessarily makes selection of series members extremely uncertain.

TABLE II

POSSIBLE SERIES IN ABSORPTION-BAND SPECTRUM OF CS_2

ν	I	$\Delta\nu$	ν	I	$\Delta\nu$	ν	I	$\Delta\nu$
26896.0	1	289.2	26648	1	285.7	26497	1	291.0
27185.2	1	283.2	26933.7	1	292.3	26788.0	1	281.5
27468.4	1	273.8	27226.0	1	272.6	27069.5	1	288.7
27742.2	1	267.7	27498.6	1	277.9	27358.2	1	.....
28009.9	1	266.7	27776.5	1	265.7	.....	.....	2·268.8
28278.6	1	266.2	28041.2	1	267.0	27895.9	1	.....
28544.8	4	266.1	28308.2	1	.....	.....	.....	2·264.0
28710.9	1	267.9	.....	.....	2·265.5	27424.0	4	264.0
28978.8	5	265.6	28839.1	16	.....	27688	1	264.6
29244.4	12	267.2	.....	.....	3·267.1	27952.6	3	.....
29511.6	7	264.6	.....	.....	.....	.....	.....	2·265.2
29776.2	23	266.3	29640.5	5	263.5	29483.1	20	.....
30042.5	23	265.8	29904.0	26	260.7	.....	.....	2·267.3
30308.3	58	267.9	30164.7	10	266.9	30017.8	15	264.5
30576.2	30	.....	30431.6	33	.....	30282.3	105	.....
.....	.....	2·265.7	.....	.....	3·266.2	.....	.....	2·265.3
31107.6	680	264.3	.....	.....	.....	30813	78	.....
31371.9	495	271.1	31230.1	36	.....	.....	.....	2·265.4
31642	175	265.8	.....	.....	2·266.2	31343.9	1000	.....
31907.8	92	264.4	31762.5	230	.....	.....	.....	3·265.7
32172.2	27	.....	.....	.....	2·264.6	.....	.....	.....
.....	.....	2·267.9	32291.6	170	.....	32140.9	40	.....
32608.1	27	265.4	.....	.....	2·267.3	.....	.....	2·266.3
32873.5	68	262.4	32826.2	30	268.7	32673.5	10	.....
33135.9	43	268.2	33094.9	46	.....	.....	.....	2·266.4
33404.1	53	267.2	.....	.....	2·265.8	33206.3	13	265.3
33671.3	13	265.1	33626.5	23	267.4	33471.6	10	.....
33936.4	10	266.9	33893.9	6	270.3	.....	.....	2·268.2
34203.3	5	.....	34164.2	5	.....	34008.1	2	.....
Means	.....	266.4	.....	.....	266.1	.....	.....	266.0

Furthermore, owing to the incompleteness of the quantum theory of triatomic molecules, one does not know just what sort of relationships to expect. Doubtless, further empirical work together with more advanced theory will ultimately lead to a satisfactorily complete analysis.

In concluding, the writer wishes to express his thanks to Dr. Alpheus W. Smith for suggestion of the problem and for helpful advice and direction; also to the Physics Department for provision of the necessary facilities.

MENDENHALL LABORATORY
OHIO STATE UNIVERSITY
June 1928

REVISION OF THE VALUE OF e/m DERIVED FROM MEASUREMENTS OF THE ZEEMAN EFFECT[1]

By HAROLD D. BABCOCK

ABSTRACT

With the aid of *recent data on the structure of the spectra of Cr and Ti* the value formerly derived for the specific *charge* of the electron has been *revised*. The previous list of 116 lines is now reduced to 48, with *only such lines* retained *as may properly be used* according to the present *theory of the Zeeman effect*. The lines now employed are those which had the greatest weight in the earlier work. *Forty-eight new values* of e/m are derived whose *weighted mean* is $e/m=(1.7606\pm0.0012)\times10^7$ E.M.U./gm. This is practically the *same as the earlier result*, but somewhat more reliable.

An earlier paper[2] gives the ratio of the elementary charge, e, to the mass, m, of an electron as determined from the observed Zeeman effect produced on 116 spectral lines by a magnetic field of measured intensity. The lines were selected from the spectra of *Cr*, *Ti*, *Zn*, *Ba*, and *Cd* between the limits λ 3841 and λ 4870, and thus only part of them could be used in any single determination of e/m. Those most frequently employed have Zeeman patterns which had been found to bear fractional relations to the normal separation involving only small denominators. The modern theory of the Zeeman effect fully confirms the observational evidence which led to the selection of these lines. Examples of this are indicated by asterisks in Table I, more fully described below. The mean result for e/m, as pointed out in *Contribution* No. 263, depends chiefly on such lines.

Occasionally, however, other lines were also used for which the theoretical Zeeman effect is unknown, and still others now shown by the theory to be unsuited to precise measurement, except with resolving power much higher than that which was then available. These additional lines were included in the investigation because, with apparently reasonable assumptions, they gave results accordant with those from better lines and because at that time the spectra involved had not been sufficiently analyzed to afford a reliable criterion for excluding unsuitable lines. Although these doubtful lines

[1] *Contributions from the Mount Wilson Observatory, Carnegie Institution of Washington*, No. 374.

[2] *Mt. Wilson Contr.*, No. 263; *Astrophysical Journal*, **58**, 149, 1923.

played a minor rôle in the investigation, their inclusion has met with criticism, and Gerlach,[1] for example, has felt that the result could be accepted as approximate only.

In recent years much progress has been made in systematizing the complex spectra. Thus *Ti* and Ti^+ have been very completely analyzed by Russell,[2] while *Cr* and Cr^+ have been studied by several investigators, among them Dr. C. C. Kiess, to whom I am particularly indebted for generous permission to use unpublished material. It has thus become possible to examine in detail the observations formerly made for the determination of e/m, and to derive a new value of this constant, based solely on lines whose Zeeman effects, as shown by theory, are capable of accurate measurement with the apparatus that was used.

It has been verbally suggested by Dr. W. V. Houston that a theoretical objection may be made to the use of the Zeeman effect for deriving e/m, because of a possible influence of adjacent spectral terms of different multiplicity on the magnetic levels of the terms involved in the measured lines. Thus a quintet D-term whose sublevels are widely separated might be affected by a triplet D-term whose distance, on the scale of wave-numbers, is not more than three or four times the separation of the extreme levels of the quintet term. In such a case spectral lines derived from combinations of the quintet with other terms would have slightly modified Zeeman effects.

Unfortunately, my data are too meager to afford a test of this question; but it appears that for the lines of greatest weight in this revision the terms are so compact, in comparison with their separation from other terms which might disturb the magnetic levels, that no influence is to be feared. For example, in the case most favorable to such an influence, the separation from possibly disturbing terms is seven times the range of wave-numbers in the term involved in a measured line.

Table I gives the 48 spectral lines which have been retained, together with the combination of terms for each line and the theoretical Zeeman pattern, which in every case is confirmed within the

[1] Geiger und Scheel, *Handbuch der Physik*, **22**, 58, Berlin, 1926.

[2] *Mt. Wilson Contr.*, Nos. 344, 345; *Astrophysical Journal*, **66**, 283, 347, 1927.

TABLE I

SPECTRAL LINES USED FOR DETERMINATION OF a

λ	El.	Multiplet	Separations
3852.23.....	*Cr*	$a^5D_1-a^3P_0$	(0) 1.50
3857.63.....	*Cr*	$a^5P'_1-d^5D'_1$	(1) 1.50, 2.50
3883.30.....	*Cr*	$a^5D_2-a^5D'_3$	(0) 1.50
3885.22.....	*Cr*	$a^5D_1-a^5D'_2$	(0) 1.50
3886.80.....	*Cr*	$a^5D_3-a^5D'_4$	(0) 1.50
3894.05.....	*Cr*	$a^5D_0-a^5D'_1$	(0) 1.50
3902.95.....	*Cr*	$a^5D_2-a^5D'_2$	(0) 1.50
3908.76.....	*Cr*	$a^5D_3-a^5D'_3$	(0) 1.50
3916.25.....	*Cr*	$a^5D_1-a^5D'_0$	(0) 1.50
3919.17.....	*Cr*	$a^5D_4-a^5D'_4$	(0) 1.50
3921.03.....	*Cr*	$a^5D_2-a^5D'_1$	(0) 1.50
3928.65.....	*Cr*	$a^5D_3-a^5D'_2$	(0) 1.50
3941.50.....	*Cr*	$a^5D_4-a^5D'_3$	(0) 1.50
3992.85.....	*Cr*	$a^5P'_3-f^5P_3$	(0) 1.67
4126.52.....	*Cr*	$a^5G_6-b^5G'_6$	(0) 1.33
4163.63.....	*Cr*	$a^5G_4-b^5G'_4$	(0) 1.15
4191.27.....	*Cr*	$a^5G_3-b^5G'_3$	(0) 0.92
4289.72.....	*Cr*	$a^7S_3-a^7P_2$	(**0**, 0.33, 0.67) **1.33**, 1.67, 2.00, 2.33, 2.67
4295.79.....	*Cr*	$a^5P'_3-e^5P_3$	(0) 1.67
4337.57*.....	*Cr*	$a^5D_1-a^5F_2$	(**0**, 0.5) **0.50**, 1.00, 1.50
4339.45.....	*Cr*	$a^5D_2-a^5F_3$	(**0**, 0.25, 0.50) **0.75**, 1.00, 1.25, 1.50, 1.75
4351.06.....	*Cr*	$a^5D_1-a^5F_1$	(1.50) 0, 1.50
4359.63*.....	*Cr*	$a^5D_2-a^5F_2$	(0.5, **1.0**) 0.50, 1.0, 1.5, 2.0
4371.28.....	*Cr*	$a^5D_3-a^5F_3$	(0.25, 0.50, **0.75**) 0.75, 1.0, **1.25**, **1.50**, 1.75, 2.00
4391.76.....	*Cr*	$a^5D_3-a^5F_2$	(**0**, 0.5, 1.0) 0.50, 1.00, 1.5, 2.0, **2.5**
4496.86*.....	*Cr*	$a^5S_2-b^5P_3$	(**0**, 0.33, 0.67) **1.0**, 1.33, 1.67, 2.0, 2.33
4526.48.....	*Cr*	$a^5G_6-a^5G'_6$	(0) 1.33
4533.25.....	*Ti*	$a^5F'_5-b^5F_5$	(0) 1.40
4554.04*.....	*Ba*+	$a^2S_1-a^2P_2$	(0.33) **1.0**, 1.67
4580.06*.....	*Cr*	$a^5S_2-b^5P_1$	(**0**, 0.5) **1.5**, 2.0, 2.5
4591.41.....	*Cr*	$a^5D_1-b^5P_2$	(**0**, 0.33) 1.5, 1.83, **2.16**
4613.34*.....	*Cr*	$a^5D_0-b^5P_1$	(0) 2.5
4616.13*.....	*Cr*	$a^5D_2-b^5P_2$	(0.33, **0.67**) 1.17, **1.50**, **1.83**, 2.16
4626.19*.....	*Cr*	$a^5D_1-b^5P_1$	(1.0) 1.50, 2.50
4651.30*.....	*Cr*	$a^5D_2-b^5P_1$	(**0**, 1.0) **0.50**, 1.5, 2.5
4652.16*.....	*Cr*	$a^5D_3-b^5P_2$	(**0**, 0.33, 0.67) **0.83**, 1.17, 1.50, 1.83, 2.16
4654.74.....	*Cr*	$a^7F_0-c^7D_1$	(0) 3.0
4678.15.....	*Cd*	$a^3P_0-a^3S_1$	(0) 2.0
4680.14*.....	*Zn*	$a^3P_0-a^3S_1$	(0) 2.0
4689.39.....	*Cr*	$a^7F_3-c^7D_2$	(**0**, 0.5, 1.0) **0.5**, 1.0, 1.5, 2.0, 2.5
4700.62.....	*Cr*	$a^5P'_1-b^7D'_2$	(**0**, 0.5) **1.5**, 2.0, 2.5
4722.16*.....	*Zn*	$a^3P_1-a^3S_1$	(0.5) 1.5, 2.0
4758.13.....	*Ti*	$a^3H'_5-c^3H_5$	(0) 1.03
4759.28.....	*Ti*	$a^3H'_6-c^3H_6$	(0) 1.17
4799.91.....	*Cd*	$a^3P_1-a^3S_1$	(0.5) 1.5, 2.0
4810.53*.....	*Zn*	$a^3P_2-a^3S_1$	(**0**, 0.5) **1.0**, 1.5, 2.0
4824.13.....	*Cr*+	$a^4F'_5-a^4F_5$	(0) 1.33
4848.27.....	*Cr*+	$a^4F'_4-a^4F_4$	(0) 1.24

* Used on ten or more photographs.

errors of observation. Close lines like λ 4651.50 and λ 4652.16 of *Cr* were used with precautions to avoid errors of measurement caused by blending of their components. Usually the outermost components, and sometimes those of intermediate separation, were omitted, depending on circumstances such as strength of magnetic

TABLE II

Plate	*a*	*H*	*e/m*	Wt.	Plate	*a*	*H*	*e/m*	Wt.
1.....	1.352	29,030	1.7543	1	28....	1.234	26,500	1.7542	2
2.....	1.348	29,030	1.7491	1	29....	1.228	26,500	1.7457	1
3.....	1.360	29,030	1.7647	1	30....	1.225	26,500	1.7414	2
4.....	1.274	27,670	1.7344	1	31....	1.228	26,500	1.7457	2
5.....	1.287	27,670	1.7521	1	32....	1.233	26,500	1.7528	2
6.....	1.284	26,950	1.7946	1	33....	1.236	26,500	1.7571	1
7.....	1.281	26,950	1.7904	1	34....	1.241	26,500	1.7642	1
8.....	1.385	29,360	1.7769	1	35....	1.145	24,290	1.7758	2
9.....	1.423	30,300	1.7690	1	36....	1.145	24,290	1.7758	2
10.....	1.411	30,300	1.7542	1	37....	1.139	24,290	1.7664	2
11.....	1.306	27,660	1.7786	1	38....	1.238	26,500	1.7599	1
12.....	1.341	28,320	1.7837	1	39....	1.236	26,500	1.7571	2
13.....	1.485	31,810	1.7582	3	40....	1.240	26,500	1.7622	2
14.....	1.498	31,810	1.7740	3	41....	1.238	26,500	1.7599	2
15.....	1.487	31,810	1.7610	2	42....	1.231	26,500	1.7500	2
16.....	1.478	31,810	1.7502	2	43....	1.232	26,500	1.7514	2
17.....	1.490	31,810	1.7646	1	44....	1.231	26,500	1.7500	2
18.....	1.474	31,810	1.7456	3	45....	1.242	26,500	1.7656	1
19.....	1.502	31,810	1.7787	3	46....	1.242	26,500	1.7656	1
20.....	1.477	31,810	1.7491	2	47....	1.242	26,500	1.7656	1
21.....	1.486	31,810	1.7598	3	48....	1.248	26,500	1.7741	1
22.....	1.487	31,810	1.7611	3					
23.....	1.490	31,810	1.7646	2					
24.....	1.487	31,810	1.7611	3					
25.....	1.490	31,810	1.7646	1					
26.....	1.488	31,810	1.7622	2					
27.....	1.244	26,500	1.7684	2					

field, spectroscopic resolving power, and dispersion. Further, the inner *p*-components of lines like λ 4339.45 of *Cr* were omitted because their small separation precludes accuracy in the result.

Table II presents the new weighted mean values of the normal separation, *a*, derived for each plate from lines contained in Table I by dividing the observed quantity $\Delta\nu$, corresponding to each pair

of Zeeman components, by the number which represents the ratio of the given separation to that of the normal triplet, as listed in the last column of Table I. Weights of individual values of a depend on the magnitude of $\Delta\nu$ and on the quality of the spectral lines. The third column of Table II gives the same magnetic intensities, H, as were found in the previous investigation, while the fourth column contains the revised values of e/m.

No new measurements are given in this paper, but the new weights in the last column of Table II have been assigned to the separate determinations of e/m because of changes in the relative number of lines employed. As shown in the previous paper, however, the weights have very little effect on the final result.

The weighted mean value of e/m is found from Table II to be 1.7610×10^{7}, which is the same as the result derived in *Contribution* No. 263. A further slight correction is required, however, because the velocity of light in air was formerly taken to be 2.9978×10^{10} cm/sec., whereas the new result of Professor Michelson[1] is 2.99704×10^{10} cm/sec. Applying this correction, we find the final value to be

$$e/m=(1.7606\pm0.0012)\times10^{7}\ \text{E.M.U./gm.}$$

In view of the selection of material on which this result rests, the probable error which is appended is of greater significance than in the former paper. It seems probable that the actual uncertainty in the result is not much greater than one part in one thousand.

The value of e/m found here coincides with that derived by Dr. Houston[2] from measurements of the spectra of hydrogen and helium. This coincidence is undoubtedly accidental, but the fact that the two investigations agree in giving results distinctly lower than those found by other observers is perhaps significant. Discussion of the theoretical questions involved will not be attempted here, but it may not be superfluous to recall that Birge,[3] in a thorough examination of the relations among basic constants, concludes that the most probable value of e/m is 1.760×10^{7} E.M.U./gm. Furthermore, Dr. Birge, at whose suggestion this revision has been made, permits me

[1] *Annual Report of the Director, Mount Wilson Observatory*, 1927.

[2] *Physical Review*, **30**, 608, 1927. [3] *Science*, **64**, 180, 1926.

to quote as follows from a paper which he is now preparing on the most probable values of general physical constants:

> Summarizing the results we find
>
> $e/m = 1.769 \pm 0.002$ from deflection experiments
> $= 1.761 \pm 0.002$ from Zeeman effect
> $= 1.761 \pm 0.001$ from *H* and *He* spectra.
>
> The discrepancy between the first result and the last two seems to be real.

Further details and the interesting conclusions reached must await the publication of Birge's paper.

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October 31, 1928

ON METEORIC MATTER NEAR THE STARS

By HENRY NORRIS RUSSELL

ABSTRACT

It has generally been assumed that solid meteorites may fall into the sun. Calculation shows, however, that *masses* of stone or iron will be *completely volatilized* by the sun's heat, before they reach its surface, unless they were originally a foot or more in diameter. The *gas* resulting from the volatilization *will*, however, proceed with unaltered speed, and *fall into the sun unless repelled by radiation pressure.*

Adopting Shapley's estimate that 1000 million meteors strike the earth every day, and Kapteyn's determination of the total quantity of matter per cubic light year in interstellar space, it is shown that the *average mass per meteorite cannot exceed 2 mg*, and that the *total* quantity of meteoric *matter* falling into the sun *is, at most, 60 tons per second.* These values hold for meteors of *interstellar* origin. If the observed meteors belong to the solar system, they may be larger, but few will hit the sun.

The total amount of light which can be scattered by the meteoric gas depends on the ratio of radiation pressure to gravity. If this is small, little light is scattered; if great, the gas is driven away from the star. The maximum effect is produced when radiation pressure and gravity nearly balance each other. *Meteoric matter near* the *sun* may scatter enough light to account for a *small fraction of the brightness of the corona*, but *cannot exert* enough *effective absorption* in the spectrum to produce the equivalent of a single narrow Fraunhofer line.

For the stars in general, conditions are similar, and it is concluded that meteoric matter, or gas arising from its volatilization, can account for perceptible bands in the spectrum only in the case of a star immersed in very dense nebulosity, and probably not even then.

The interesting suggestion has recently been made by Dr. Shapley and Miss Payne[1] that certain wide, shallow, and diffuse absorption bands in stellar spectra are produced by meteoric matter falling into the stars, or moving near them with high velocity. The present paper attempts a quantitative discussion of some of the problems involved. In the nature of the case, the discussion must deal mainly with the order of magnitude of the effects to be expected.

1. VAPORIZATION OF METEORITES NEAR A STAR

It is generally assumed that meteorites may actually collide with the sun—retaining their solid state until the moment of entering its atmosphere. It may be shown, however, that all but large bodies will be volatilized before they reach the surface.

The best chance for survival will evidently be possessed by a meteor which falls straight into the sun, and spends the least possible time in its neighborhood before the impact. At the distances which

[1] *Harvard Circular*, No. 317, May, 1928.

need be considered, the meteor's velocity will be due almost wholly to the sun's attraction. We may therefore safely assume that it is traveling with the parabolic velocity. The time t of fall from the distance r to the surface is

$$t = A\left\{\left(\frac{r}{R}\right)^{\frac{3}{2}} - 1\right\}, \tag{1}$$

where R is the sun's radius. The constant A is 750 sec. for the sun. For other stars it varies as $M^{-\frac{1}{2}}R^{\frac{3}{2}}$. A small conducting black sphere, in thermal equilibrium with the solar radiation at the distance r, will take up a temperature T defined by the equation

$$T = B\left(\frac{R}{r}\right)^{\frac{1}{2}}. \tag{2}$$

The constant B is $1/\sqrt{2}$ times the effective temperature of the star. and, for the sun, is approximately 4050°.

Substituting, we have

$$t = A\left(\frac{B^3}{T^3} - 1\right). \tag{3}$$

If the meteorite, as it falls into the sun, keeps always the temperature T appropriate to its distance, its temperature will rise at the rate

$$\frac{dT}{dt} = \frac{T^4}{3AB^3}. \tag{4}$$

Suppose now that the meteorite is a sphere of radius x, density ρ, and specific heat C. Its heat capacity in calories will be $\frac{4}{3}\pi\rho Cx^3$ and the rate of supply of energy, in ergs per second, necessary to maintain this rise of temperature, will be

$$\frac{dQ}{dt} = \tfrac{4}{3}\pi\rho JCx^3\,\frac{T^4}{AB^3}, \tag{5}$$

where J is the mechanical equivalent of heat.

At the same time, the body will be radiating heat from its surface at a total rate

$$\frac{dQ'}{dt} = 4\pi\sigma x^2 T^4, \tag{6}$$

where σ is Stefan's constant. Hence

$$\frac{dQ}{dQ'}=\frac{J\rho Cx}{9\sigma AB^3} \tag{7}$$

and is independent of the distance from the sun, but proportional to the radius of the body.

Introducing $J=4.18\times10^7$, $\sigma=5.72\times10^{-5}$, we find for the sun

$$\frac{dQ}{dQ'}=1.63\times10^{-3}\rho Cx\ . \tag{8}$$

For a stony meteorite we may take $\rho=3$, $C=0.25$,[1] whence $dQ/dQ'=x/815$; for one of iron, $\rho=7.8$, $C=0.15$, $dQ/dQ'=x/530$.

Since a part of the heat received from the sun must be spent in warming the meteorite, its temperature can never quite reach the value T (provided, of course, that the thermal conductivity is very great and the internal temperature uniform). If the actual temperature is fT, the heat lost by radiation must be multiplied by f^4, and that taken up by the body by f. The heat losses are then $f\frac{dQ}{dt}+f^4\frac{dQ'}{dt}$, while the gain from the sun is dQ'/dt as before. Equating these, we find

$$f^4+\frac{\rho Cx}{614}f=1\ , \tag{9}$$

whence

$f=$	1.00	0.99	0.95	0.90	0.80	0.70	0.60	0.50
$\rho Cx=$	0	25	120	236	454	670	890	1100

The effects of the heat capacity of the mass may, therefore, be neglected except for bodies a meter or more in diameter.

For such bodies, the assumption of an instantaneous distribution of heat within the mass is grotesquely wrong, and our approximation fails.

If the supply of solar heat should suddenly be cut off from the

[1] The observed specific heats at high temperatures are here used.

body (still supposed to be perfectly conducting), the equation of cooling will be

$$\frac{dT}{dt} = -\frac{3\sigma T^4}{J\rho C x} = 4.10 \times 10^{-12} \frac{T^4}{\rho C x}.$$

If the initial temperature is T_0 and we set $t_0 = T_0/(dT/dt)_0$, the integral is $(T/T_0)^3 = t_0/[t_0 + 3(t - t_0)]$. The temperature will fall to half its initial value in the time $\frac{7}{3}t_0$. In the present case this is $(5.69 \times 10^{11})/T_0^3 \cdot \rho C x$. For $T = 2000°$, $2500°$, and $3000°$, these times, in seconds, are $71\rho Cx$, $37\rho Cx$, and $21\rho Cx$, respectively. Since ρC is of the order of unity, the smaller meteorites would cool very rapidly, even if all the internal heat escaped at once.

The actual surface cooling would be still more rapid. The thermal conductivity of iron is 0.15, of rock about 0.005, in calories per square centimeter for a temperature gradient of 1° per centimeter. To supply the loss of heat from a black surface at 1000°, the gradient in iron would be 9° per centimeter; for a surface at 2000°, 145°/cm, and at 3000°, 740°/cm. In rock the gradients would be about thirty times greater. It follows, then, unless the meteorite is in rapid rotation, that its surface temperature must be very different on the sunlit and dark sides. Conduction of heat will be of importance only in masses of iron not more than a few centimeters in diameter, or of rock a very few millimeters across. For larger bodies the temperature at the middle of the sunlit side will approach the value which it would have if it permanently faced the sun. This is $1/\sqrt{2}$ times the value of T defined by (2).

We will, however, for the present, return to the simplified case of the highly conducting body of uniform temperature. As it approaches the sun, its temperature will rise until it reaches the melting-point. With a slight pause due to the latent heat of fusion, it will rise to the boiling-point. (We are here assuming that the surface of the liquid remains "black.") The actual onset of evaporation, *in vacuo*, will be gradual. To simplify the calculation, we will assume that evaporation begins abruptly at a temperature E, and that the solid or liquid mass cannot rise above this temperature. As the body approaches the sun, its surface will continue to radiate heat at the rate corresponding to E, and the excess solar heat which it receives

will be employed in evaporating its substance. The rate of reception of this excess heat is $4\pi\sigma x^2(T^4-E^4)$. If L is the latent heat of evaporation, the rate at which the meteorite loses mass will be

$$\frac{dm}{dt}=-\frac{4\pi\sigma x^2}{JL}(T^4-E^4)\,.$$

Transforming to T as independent variable and expressing m in terms of x, we find, after simple reductions,

$$\frac{dx}{dT}=\frac{3AB^3\sigma}{\rho JL}\left(\frac{E^4}{T^4}-1\right)\,. \tag{10}$$

All meteorites of the same composition, at the same distance from the sun, will therefore diminish in diameter at the same rate—which accords with the fact that the volatilization is a surface effect. Integrating, we find

$$\left.\begin{aligned} x_0-x&=\frac{AB^3\sigma}{\rho JL}\left(3T+\frac{E^4}{T^3}-4E\right)\,,\\ &=\frac{AB^3\sigma}{\rho JL}(T-E)^2(3T^2+2TE+E^2)/T^3\,. \end{aligned}\right\} \tag{11}$$

Introducing numerical values, and setting $T=E\,(1+y)$, we have

$$x_0-x=\frac{67\cdot 9}{\rho L}E\,\frac{6y^2+8y^3+3y^4}{(1+y)^3}\,.$$

The latent heat of evaporation of metals is of the order of 1000 calories per gram.[1] The coefficient of E is therefore of the order of 0.01. Adopting this value, we find, if $y=0.1$,

$$x_0-x=5\times 10^{-4}T_0\,.$$

If T_0 is 2000° (a reasonable value), a layer 1 cm thick will be evaporated off the meteorite by the time it reaches the distance where the equilibrium temperature is 2200°. When this temperature is 2400°, the thickness removed comes out 3.5 cm, when $T=3000°$ it is 30 cm, and for $T=4000°$, just outside the sun's surface, 43 cm.

For a poorly conducting body, volatilization will begin earlier, and the excess heat available for it will be greater at a given distance, so that the total loss by vaporization will be greater. If the meteor

[1] See *Smithsonian Physical Tables* (7th ed.), p. 233, 1920.

does not behave like a black body, we may assume it to be "gray." In this case the rates of absorption and radiation of heat, and of volatilization, will all be reduced in the same ratio. With an effective albedo of 0.5 (a very high value) the loss of diameter would be one-half of the amount calculated above. It appears, therefore, that masses of rock or metal cannot persist in solid or liquid form up to the sun's surface, even under the most favorable conditions of approach, unless their initial diameters are of the order of a foot or more. All smaller ones will be volatilized before they strike the surface.

The diminution of size by evaporation, on reaching the surface of any star, is found by setting $T=B$ in (11). If E is small compared with B (as is true in the most important cases), this will vary as AB^4, i.e., as $\rho^{-\frac{1}{3}}T_e^4$ when ρ is the star's density and T_e its effective temperature.

The evaporation of meteors should therefore be particularly effective in the vicinity of the hotter stars.

The idea of solid meteorites plunging into the sun's atmosphere must therefore be abandoned, except in the case of large masses. The gaseous débris of the infalling smaller bodies, however (unless held back by radiation pressure), will keep on with the parabolic velocity, and strike the upper chromosphere as a shower of swiftly moving individual atoms. The kinetic energy of a single hydrogen atom under these circumstances equals that of an electron which has fallen through 1985 volts. Atomic collisions of great violence may therefore be expected, and much of the energy may go into ionization and excitation of the chromosphere.

It may be noted in passing that the great comet of 1882 passed, at perihelion, through a region where T was about 3000°. Its nucleus, which survived the passage, though breaking into four parts, must have been composed of masses of considerable size. As this comet is undoubtedly periodic, it must have been stripped of all finer material at earlier returns, and is probably far from typical of other comets.

2. PARTICLES UNDER CENTRAL FORCE

We have occasion to use some properties of the distribution of particles moving in the vicinity of a star. Many of these may be

proved as simply for the case of a generalized field of central force. The results are doubtless not new, but it is convenient to give them here.

Suppose that particles are moving in a steady state, in a field of force of the form $mF(r)$, where r is the distance from the center. If v is the velocity and θ the angle between v and r, the first integrals of motion are

$$v^2 = U(r) + C \; ; \qquad rv \sin\theta = h \; ,$$

where U is the potential and C and h are constants. Suppose further that at some given distance r_0 the particles are distributed with uniform density d_0 (particles per unit volume), and are moving with a given velocity v_0, and isotropically as to direction. The number of particles which pass through unit area of a sphere of radius r_0 in unit time and at angles between θ_0 and $\theta_0 + d\theta_0$ with the radius will be

$$\tfrac{1}{2} d_0 v_0 \cos\theta_0 \sin\theta_0 d\theta_0 \; .$$

The whole number passing through the r_0-sphere within these limits of angle will be

$$N(r_0, \theta_0) = 2\pi d_0 r_0^2 v_0 \cos\theta_0 \sin\theta_0 d\theta_0 \; . \tag{12}$$

After traveling to some other distance r, these particles will all have the same velocity v, and move at angles lying between θ and $\theta + d\theta$, where

$$\left.\begin{aligned} &v^2 - v_0^2 = U(r) - U(r_0) \; , \\ &rv \sin\theta = r_0 v_0 \sin\theta_0 \; , \\ &rv \cos\theta d\theta = r_0 v_0 \cos\theta_0 d\theta_0 \; . \end{aligned}\right\} \tag{13}$$

If any of the particles reach the r-sphere, all will do so, and the whole number, $N(r,\theta)$, crossing this sphere in unit time will be the same as for the r_0-sphere.

From (12) we find

$$N(r, \theta) = 2\pi \frac{d_0 v}{v_0} r^2 v \cos\theta \sin\theta d\theta \; .$$

This is the same as would be found for an isotropic distribution at distance r with density $d_0(v/v_0)$.

Two cases may now be distinguished:

I. If $r_0v_0 \geqq rv$, only those particles which pass through the r_0-sphere (inward or outward) at angles such that $\sin \theta_0 \leqq rv/r_0v_0$ will cross the r-sphere. For these particles θ will range from 0 to π. The distribution of velocities at the distance r will be isotropic.

II. If $r_0v_0 < rv$, the particles which have crossed the r_0-sphere will all cut the r-sphere at angles such that $\sin \theta < r_0v_0/rv$. If we set

$$\frac{r_0v_0}{rv} = \sin \varphi \,, \tag{14}$$

θ must be between 0 and φ, or between $\pi - \varphi$ and π. Within these limits the particles come in equal numbers from all directions. What happens outside the limits is not yet specified.

We will now suppose that for these values of θ there are no particles present; i.e., that the r_0-sphere marks the boundary of the region of isotropic distribution of velocities. On one side of this sphere case I will apply and the velocity distribution will be isotropic, and the density will be

$$d = d_0v/v_0 \,, \tag{15}$$

and on the other side the velocities will all be directed within a cone of semi-angle φ. The space density d of the particles at distance r will then be d_0v/v_0 times the ratio of the solid angle contained within this cone to the sphere; that is,

$$d = \frac{d_0v}{v_0}(1 - \cos \varphi) \,. \tag{16}$$

The number of particles moving inward across the r-sphere, and the space density of such particles, will be half as great.

Suppose now that the particles are coming in uniformly from all directions, with velocity u and space density D, at a great distance. The velocity distribution will then be isotropic at all points. At the distance r the velocity will be given by

$$v^2 = U(r) + u^2 \tag{17}$$

and the space density by

$$d = \frac{vD}{u} \,. \tag{18}$$

3. METEORS AND GAS NEAR A STAR

In applying these results to meteors passing near a star, we must consider three complications: (*a*) Those meteors which come near the star will be volatilized. (*b*) A part of the resulting gas will fall into the star. (*c*) The remainder of the gas may scatter the star's radiation, and be affected by radiation pressure. The effects of volatilization and radiation pressure may vary widely with the composition; but it is evidently sufficient to work them out for meteors of uniform composition, and then, if necessary, to superpose the effects for those of different nature.

The reasoning of paragraph 1 indicates that no serious error will be committed if we assume that all meteors which approach the star within a certain limiting distance r_0 and reach the evaporation temperature E are forthwith converted into gas. From (2) we have

$$r_0 = RB^2/E^2 \,. \tag{19}$$

Outside this sphere the velocity of the meteors will be given by

$$v^2 = \frac{2GM}{r} + u^2 \,, \tag{20}$$

M being the star's mass and G the constant of gravitation. At any given point, they will appear to come uniformly from all points of the celestial sphere except from those within an elongation φ from the star, where $\sin \varphi = r_0 v_0 / rv$ (14). The density of distribution of the meteors is then

$$d = \frac{vD}{2u} (1 + \cos \varphi) \,. \tag{21}$$

Inside the critical sphere, the meteors are replaced by gas. We will assume that the force due to radiation pressure is K times that of gravity. We then have for the velocity of the particles

$$v^2 = \frac{2(1-K)GM}{r} + \frac{2KGM}{r_0} + u^2 \,. \tag{22}$$

The gas molecules at any point will come from all directions except those at an elongation less than ψ, where

$$\sin \psi = RV/rv \,. \tag{23}$$

The density of the gas will be given by the equation

$$\rho=\frac{mvD}{2u}\,(1+\cos\psi)\,, \tag{24}$$

where m is the average mass of a meteor. Outside the critical sphere, the gas molecules are all traveling from directions lying between the elongations φ and ψ, and the density of the gas is

$$\rho'=\frac{mvD}{2u}\,(\cos\psi-\cos\varphi)\,.$$

Here φ is still to be defined by (14), but v by (22) instead of (20). Note that

$$\rho-\rho'=\frac{mvD}{2u}\,(1+\cos\varphi)\,. \tag{25}$$

It represents the deficiency of gas outside the critical sphere due to unvolatilized meteors.

If the effects of collisions with the star and of radiation pressure are negligible, the density of the gas just inside the critical sphere is equal to the sum of those due to the gas and meteors just outside, which are equal to one another. When these influences are sensible, the density of the gas is less. If the radiation pressure exceeds gravity, the gas may all be turned back before it reaches the star's surface, and the terms in ψ disappear from the equations.

The whole number, N', of meteors which fall in unit time upon a fixed sphere of radius R' at distance r from the star will be

$$N'=\pi dR'^2v=\frac{\pi R'^2D}{2u}\,(1+\cos\varphi)\,\left(\frac{2GM}{r}+u^2\right)\,. \tag{26}$$

Here the effect of the attraction of the sphere itself has been neglected. It may be taken into account by adding to the last parentheses the quantity $2GM'/R'$, where M' is the mass of the sphere.

The mass of gas which falls into the star in unit time will be

$$M'=\pi R^2\,\frac{mD}{u}\,V^2=\frac{\pi R^2mD}{u}\,\left\{2GM\left(\frac{1-K}{R}+\frac{K}{r_0}\right)+u^2\right\}\,. \tag{27}$$

This is twice as great in proportion to the area as for a small sphere just outside the star's surface, because, in this region, all the molecules are moving inward.

No gas will reach the star if the radiation pressure exceeds the value for which the parentheses in (27) vanishes.

By (19) the limiting condition becomes

$$(B^2-E^2)K=B^2(1+u^2R/2GM) .$$

The term $u^2R/2GM$ is usually negligible. The ratio of radiation pressure to gravity is, then,

$$K=B^2/(B^2-E^2) . \tag{28}$$

The whole mass of gas within the critical sphere will be

$$\mu=4\pi\int_R^{r_0}\rho r^2dr=2\pi\frac{mD}{u}\int_R^{r_0}(1+\cos\psi)vr^2dr .$$

By the theorem of mean value, we may set

$$\mu=2\pi\frac{mD}{u}(1+\cos\psi m)\int_R^{r_0}r^2vdr ,$$

where ψm is a suitable mean value of ψ. Neglecting the difference between the parabolic and hyperbolic velocities inside this sphere, we have, when $K=0$, $v=\sqrt{2GM/r}$, and, when $K=1$, $v=\sqrt{2GM/r_0}$.

If we set

$$\rho_0=\frac{mv_0D}{u}=\frac{mD}{u}\sqrt{2GM/r_0} ,$$

we find

$$\mu_0=\tfrac{8}{5}\pi\rho_0\frac{1+\cos\psi m}{2}(r_0^3-R^{\frac{3}{2}}r_0^{\frac{1}{2}})$$

$$\mu_1=\tfrac{4}{3}\pi\rho_0\frac{1+\cos\psi m}{2}(r_0^3-R^3) . \tag{29}$$

The first of these is nearly six-fifths of the second. The concentration of the gas toward the star by its gravitation therefore increases the total amount within the sphere by only 20 per cent. Repulsion by strong radiation pressure would diminish it seriously.

4. SCATTERING OF LIGHT BY THE GAS

The molecules or atoms of the gas will scatter the incident radiation from the star, both by ordinary non-selective scattering and by

resonance. Some of the scattered radiation will return into the star, and the rest will escape into space.

At this point an important difference must be noted between the phenomena perceptible by an observer near the star and one at an interstellar distance. In the case of ordinary scattering the near observer will see the star surrounded by a luminous envelope, resembling the solar corona, and giving a (substantially) continuous spectrum. For the distant observer, this will be merged in the diffraction image of the star, and nothing will be observable. Selective scattering will deplete certain wave-lengths, and produce one or more dark spectral lines, which, if the gas were at rest, would be sharp, but, owing to its rapid motion, will be spread out into diffuse bands, as suggested by Shapley. The scattered light will appear to the near observer in the form of a corona about the star, giving a spectrum of broad bright bands. For the distant observer this will be fused with the star image, and the dark bands in the stellar spectrum will be correspondingly shallowed.

The radiation which is scattered back into the star will presumably, in any case, be converted almost wholly into heat in its atmosphere, and will produce a slight increase in its general radiation, which will be wholly impossible of detection. Any portion of it which is re-radiated selectively will operate to fill up the shallow dark spectral bands still further.

Some of the scattered light will be caught by other gas molecules and rescattered. The similar process for a thin stellar atmosphere has been treated by Schuster[1] in a classic paper. The net effect is that the dark lines (or bands) though increasing slowly in intensity with increase in the amount of scattering material are not as strong as the rudimentary theory indicates.

These considerations show that the amount of the directly scattered energy, which, if unobstructed, would return into the star, sets an upper limit to the amount which may appear as depletion in the dark bands of the spectrum. With sufficient approximation, we may assume that this amount bears the same ratio to the whole amount of scattered energy that the solid angle occupied by the star, as seen from the point of scattering, bears to the whole sphere. The

[1] *Astrophysical Journal*, **21**, 1, 1905.

whole scattered energy bears a simple relation to quantities which have been considered earlier. If g is the acceleration of gravity, the gas in unit volume acquires the downward momentum ρg in unit time. If in this time it scatters the amount q of the star's radiant energy, the upward momentum communicated to it will be $(q/c)[(1+\cos s)/2]$, where s is the semidiameter of the star's disk (which we assume to be of uniform apparent brightness). By definition, the latter quantity is K times the former, whence

$$q=\frac{2Kcg\rho}{1+\cos s}, \tag{30}$$

also

$$\sin s=R/r. \tag{31}$$

It is well known that the laws of scattering, whether by solid particles or by resonant atoms, are such that the ratio k will remain the same at all distances from the star. Equation (30) may therefore be applied generally without regard to the special processes involved.

Consider now the gas contained in a sphere surrounding the star between the distances r and $r+dr$. It will scatter energy at the rate $4\pi qr^2dr$. Of this scattered radiation, the fraction $(1-\cos s)/2$ reenters the star, and this alone represents possible observable depletion. The upper limit to the energy which can actually be depleted from the spectrum is therefore $2\pi\int_R^\infty qr^2(1-\cos s)dr$. The corresponding quantity per unit area of the star's surface is

$$L=\frac{1}{2R^2}\int_R^\infty qr^2(1-\cos s)dr.$$

Introducing $g=GM/r^2$, we find easily

$$\left.\begin{aligned} L&=\frac{KcGM}{R^2}\int_R^\infty\frac{\rho(1-\cos s)}{1+\cos s}\,dr-\int_{r_0}^\infty\frac{(\rho-\rho')(1-\cos s)}{1+\cos s}\,dr,\\ &=\frac{KcGMmD}{2uR}\left\{\int_0^{\pi/2}\frac{v(1+\cos\psi)\cos s\,ds}{(1+\cos s)^2}\right.\\ &\qquad\left.-\int_0^{s_0'}\frac{v(1+\cos\varphi)\cos s\,ds}{(1+\cos s)^2}\right\}, \end{aligned}\right\} \tag{32}$$

where $\sin s_0'=R/r_0=E^2/B^2$.

This, with the equations (22), (23), and (14) for v, ψ, and φ, gives the solution of the problem. The integrals must usually be evaluated by quadratures. When $K=1$, however, v is constant, $\psi=s$, and $\sin s = \sin\varphi \sin s_0$. The first integral in (32) then has the value $(\pi/2-1)v$, while the second may be written

$$v \sin s' \int_0^{\pi/2} \frac{(1+\cos\varphi)\cos\varphi d\varphi}{(1+\cos s)^2} = \frac{1}{u} v \sin s_0' \left\{ 1+\frac{\pi}{4}+\sin^2 s_0'\left(\tfrac{1}{3}+\frac{\pi}{16}\right)\right\} .$$

In this case we have

$$L=\frac{KcGMmDv}{2uR}(0.57-0.45E^2/B^2-0.13E^6/B^6) .$$

Quadratures have been made for a typical example of the more general case, namely, when $R/r_0=0.1$, and the velocity at infinity, u, is negligible compared with the parabolic velocity in the region near the star. We may then write

$$v^2=U^2\{Kx_0+(1+K)x\}=U^2w^2 , \tag{33}$$

where $x=\sin s=R/r$, $x_0=R/r_0$, and $U=\sqrt{2GM/R}$, and is the parabolic velocity of a meteor at the star's surface. We then find

$$L=\frac{KcGMmDU}{2uR}(I_1-I_2) , \tag{34}$$

where I_1 and I_2 are obtained by setting w in place of v in the two integrals in (32).

By graphical interpolation between the values obtained by quadrature, Table I was obtained. As K increases, the gas is driven away from the star by radiation pressure, and I_1 diminishes. The change is most rapid near the value $K=1.11$ for which the molecules just fail to reach the star's surface. The integral I_2, which represents the deficiency of gas due to the fact that the incoming meteors are not volatilized, is small, and changes slowly. The capacity of the gaseous envelope to deplete the spectrum, which is proportional to $K(I_1-I_2)$, is a maximum for $K=0.85$.

As x_0 changes, and with it the distance at which volatilization occurs, the conditions alter. For our purpose it will suffice to follow them for the case $K=1$. Here the integrals are much simplified and

we have $w=\sqrt{x_0}$, $I_1=(\pi/2-1)w$. Determining I_2 by quadrature, we have Table II. The value of I_1-I_2 is a maximum for $x_0=0.4$, i.e., when the distance at which volatilization occurs is 2.5 times the star's radius. If the meteors evaporate at a greater distance, the resulting gas is spread through a larger sphere and the quantity in the vicinity of the star, where most of the spectral depletion takes

TABLE I

VALUES OF INTEGRALS WHEN $x_0=0.10$

	K										
	0.0	0.2	0.4	0.6	0.8	1.0	1.1	1.2	1.4	1.6	2.0
I_1	0.363	0.339	0.312	0.277	0.233	0.180	0.139	0.074	0.045	0.035	0.029
I_2	.008	.009	.010	.012	.013	.014	.014	.014	.015	.016	.017
I_1-I_2	.355	.330	.302	.265	.220	.166	.125	.060	.030	.019	.012+
$K(I_1-I_2)$	0.000	0.066	0.121	0.159	0.176	0.166	0.138	0.072	0.042	0.030	0.025

place, is smaller. If, however, the critical sphere is too near the star, the loss due to the presence of meteors, instead of gas, outside it more than counteracts the first effect.

TABLE II

VALUES OF INTEGRALS WHEN $K=1.0$

	x_0										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
I_1	0.000	0.180	0.256	0.312	0.361	0.404	0.442	0.477	0.510	0.541	0.571
I_2	.000	.014	.040	.074	.116	.164	.220	.286	.360	.451	.571
I_1-I_2	0.000	0.166	0.216	0.238	0.245	0.240	0.222	0.189	0.150	0.090	0.000

If w_1 is the value of w when $K=1$, it follows from (3) that

$$\frac{w^2}{w_1^2}=K+(1-K)\frac{x}{x_0}.$$

As x_0 increases, the values of w/w_1 therefore decrease if $K<1$, but increase if $K>1$. The same will be true of I_1-I_2. The maximum of $K(I_1-I_2)$ for changes in K will therefore move toward greater values of K as x_0 increases. The value 0.25 for $K=1$, $x_0=0.4$, must therefore be close to the absolute maximum for this fraction, and we can safely adopt $K(I_1-I_2)<0.3$ for all values of the variables.

We then have

$$L<0.3\,\frac{cGMmDU}{2uR}$$

$$L<0.21cG^{\frac{1}{3}}\,\frac{M^{\frac{2}{3}}}{R^{\frac{1}{3}}}\,\frac{mD}{u}<0.11\left(\frac{M}{R}\right)^{\frac{2}{3}}\frac{mD}{u}\,. \tag{35}$$

We have here an absolute upper limit to the amount of radiant energy in c.g.s. units per square centimeter of the star's surface, which can appear as depletion of the spectrum. It is independent of the nature of the spectral absorption or of the boiling-point of the meteoric material, since both of these have been adjusted so as to give optimum conditions.

5. NUMERICAL APPLICATIONS

We may now apply these results to meteors entering the solar system. Following Shapley (*loc. cit*), we will assume that the earth encounters 10^9 of them per day. This is much more than the conventional estimates, but probably much better, since they made little or no allowance for telescopic meteors. In any event, it may be accepted as "a convenient number to state." The number per second, N', is then 1.16×10^4. Setting this in (26) with $R'=6.37\times10^8$, $v=4.21\times10^6$ cm/sec. (the parabolic velocity) $\cos\varphi=1$, we find for the density of distribution near the earth's orbit, $d=2.16\times10^{-21}$ meteors/cm^2, and also $D/u=5.13\times10^{-28}$.

The corrections for the attraction of the earth, for the increase of the number encountered, owing to the earth's orbital motion, and for the probable hyperbolic velocity of the meteors, will operate to diminish d, but should not do more, at most, than to halve it. The assumption that they are not affected by radiation pressure is probably quite safe.

The number of meteors striking the sun—if they did not volatilize—would be greater in the ratio Rr/R'^2 or 2.56×10^6, and comes out 2.97×10^{10} per second. The amount of gas which actually enters the sun will be $1-\mathrm{K}+K(R/r_0)$ times that of this number of meteorites. Under the optimum conditions for spectral depletion it will be 40 per cent of it.

For the maximum depletion, we find from (35)

$$L < 2.8 \times 10^5 m \text{ ergs/cm}^2 \text{ sec.}$$

Now a black body at 5750° K—the sun's effective temperature—radiates 7.1×10^6 ergs/cm² sec. within a spectral region 1A in width, in the vicinity of λ 4000. The whole loss of energy from the spectrum due to the meteoric gases must therefore be less than the amount cut out by a single dark line in the violet, $0.04m$ A in width.

To get an idea of m_1 we may note that a cubic parsec has a volume of 2.9×10^{55} cm³. On our assumption the number of meteors within this volume would be $1.5 \times 10^{28} u$. Now u is the velocity of a meteor in interstellar space, relative to the sun, and unless they share the sun's motion in space this must be of the order of 20 km/sec. With this velocity at infinity, the velocity at the earth's distance is increased from 42 to 46.5 km/sec. Taking this into account, we find the number of meteors in a cubic parsec to be 2.7×10^{34}.

Now Kapteyn[1] finds, for the star density near the sun, which is the maximum in his generalized model of the galaxy, the value 0.0358 stars per cubic parsec, and that the whole mass per star, including dark matter in the interstellar spaces, is 2.2 times the sun's mass. This gives 1.6×10^{32} gm per cubic parsec, which is undoubtedly of the correct order of magnitude. If all the mass was in the form of meteors, we would have $m = 0.006$. At least half of this mass is accounted for by the visible stars, so that the actual value of m is probably at most 0.002. This is regarded by Shapley (*loc. cit.*) as a low estimate; but it seems to be about as high as is consistent with the hypothesis of interstellar origin. Should the estimated number N' of meteors be diminished, m would be increased, but $N'm$ remains constant.

If the meteoric matter—like the known comets—shares the sun's motion in space, a much greater average mass is permissible. Suppose, for example, that the meteors are moving in elliptical orbits with aphelion distances a, then $v^2 = 2GM(1/r - 1/a)$ and the density of distribution $d = d_1 v/v_1$, where d_1 and v_1 are the density and velocity

[1] *Ibid.*, **55**, 302, 1922.

at the earth's distance r_1. The whole number of meteors is then $N = 4\pi \int_R^a r d^2 dr$. No sensible error arises from taking the lower limit as zero, instead of the sun's radius. If, then, we set $r = a \cos^2 z$, we find $v/v_1 = \sqrt{r_1/(a - r_1)} \tan z$; neglecting r_1/a, we have

$$N = 8\pi a^3 d_1 \sqrt{r_1/a} \int_0^{\pi/2} \cos^4 z \sin^2 z \, dz = \tfrac{1}{2}\pi a^{\frac{5}{2}} r_1^{\frac{1}{2}} d_1 .$$

If a is 1 parsec, and $d = 2.16 \times 10^{-21}$, we find $N = 2.2 \times 10^{32}$ meteors within the sphere, or 5.2×10^{31} per cubic parsec. With an average mass of 1 gm, the aggregate mass of the meteors associated with the sun would be one-tenth that of the sun itself. With $m = 0.002$, it is a little less than the mass of Saturn. These figures are derived on the assumption that the motions of the meteors are isotropic in direction throughout the large sphere. If they were originally ejected from the sun, so that their orbits are highly elongated, the total number may be very much smaller, and it is possible to reconcile a high estimate of the amount of meteoric material which falls on the earth with a low density in interstellar space, even near the sun. Meteors of this sort, however, can hardly maintain a permanent gaseous envelope around the sun. A very small radiation pressure suffices to make the orbits of the gas molecules hyperbolic. The envelope must therefore be continually fed by incoming meteors. Now the period of a meteor with an aphelion distance of a parsec is 32,000,000 years. In something like this interval all the meteors with perihelia lying within the critical sphere would enter it and be volatilized, leaving for a future supply only the small number which were deflected toward the sun by planetary perturbations. Only meteors of interstellar origin, therefore, can now be available to keep up the sun's gaseous envelope—though meteors belonging to the solar system, in the extended sense here described, might account for a large majority of those which strike the earth. Whether this is actually the case must be determined by observation. If Hoffmeister's conclusion that most of the sporadic naked-eye meteors have hyperbolic velocities can be extended to telescopic meteors, the low value of m derived above stands good. If only a minority of the observed mete-

ors are of "cosmic" origin, the permissible estimate of their average mass is correspondingly increased; but the density of their distribution in space is *pari passu* diminished. Our conclusion regarding the amount of depletion by meteoric matter depends, indeed, only on Kapteyn's estimate of the mean density of the galactic system, and our estimate of the velocity u of the sun relative to the meteors. There appears, therefore, to be no escape from the conclusion that the whole depletion of energy in the spectrum, due to gas of meteoric origin near the sun, can at most be enough to make a single dark line 0.0001 A wide. Taking for the radius of the critical sphere the optimum value $2.5R$, the density of the gas just inside it is found to be $2\times10^{-20}\,m$ gm/cm^3. By (29) the whole mass inside this sphere is less than $5.4\times10^{14}m$ gm or than 1.1×10^{12} gm, with the maximum permissible value of m. According to the usual estimate, the pressure at the base of the reversing layer is of the order of 100 dynes/cm^2. This makes the mass of the reversing layer 3.7×10^{-3} gm per square centimeter of the sun's surface, and its total mass 2.2×10^{20} gm, which is 200,000,000 times that of the gaseous envelope. It is no wonder, then, that the latter produces no perceptible effect on the spectrum.

Though gas of meteoric origin cannot produce any sensible effect on the solar spectrum, it may make some small contribution to the light of the corona. The observed brightness of the corona is of the order of 2×10^{-6} that of the sun. Allowing for the parts in front of the sun and behind it, its whole radiation may be 10^{-5} of the sun's, or 6×10^5 ergs/sec. for each square centimeter of the sun's surface. The maximum depletion by meteoric gas, if $m=2\times10^{-3}$, is 6×10^2 ergs/cm^2 sec. The amount of coronal radiation which escapes to space is more than that which re-enters the sun, but certainly not ten times as much—considering that most of the light comes from the inner corona. The contribution of meteoric gas to the coronal light must then be at most a fraction of 1 per cent. This conclusion is confirmed by the presence of sharp detail in the corona and of the Fraunhofer lines in its spectrum, which show that most of the light comes from slowly moving sources. With $m=0.002$, the amount of meteoric matter falling into the sun per second comes out 60 tons per second, making no deduction for diminution by the effects of

radiation pressure. To make up the loss of mass by radiation would require a meteoric density 70,000 times as great.

The conditions for the appearance of absorption bands of meteoric origin in stellar spectra are but little more favorable. By (35) the depletion (other things being equal) varies from star to star as $(M/R)^{\frac{1}{3}}$. It is well known that this quantity is of the same order of magnitude for all the stars of the main series. For typical stars of class A and B, M/R is rarely more than twice as great as for the sun.[1]

This may increase the value of L threefold, and a similar factor of increase might be secured by the assumption that the space velocity of the star relative to the meteors was 7 instead of 20 km. The combined gain is, however, offset by the fact that our formula gives L in ergs/cm^2 sec., while the photographic surface brightness of an A star is fully ten times that of the sun. The increased depletion will thus produce no more conspicuous spectral features than before.

It appears, therefore, that in any region where the density of distribution of meteoric matter does not exceed the central density of the "Kapteyn universe," the maximum depletion of a stellar spectrum, due to gas arising from the volatilization of incoming meteorites, cannot exceed the amount necessary to produce a single black line 0.0001 A wide. The observed depressions in the spectral energy curves are of the order of 200 A wide, with a depth corresponding to a loss of 5 per cent of the energy in this region. To produce such an effect, even under optimum conditions, the density of the meteoric cloud would have to be 100,000 times the average density with which matter is distributed in interstellar space.

At this higher density (5×10^{-19} gm/cm^3) the amount of matter in a column 1 cm^2 in cross-section and 1 parsec long would be 1.5 gm. Unless the meteors were practically all as large as 1 cm in diameter, such a cloud would be opaque in a thickness of a parsec. The meteoric cloud would, therefore, be a dark nebula. The writer's rough estimate[2] that a dark nebula 1 parsec in diameter contains

[1] See, e.g., Russell, Dugan, and Stewart, *Astronomy*, pp. 719 and 740. Boston: Ginn & Co., 1927.

[2] *Proceedings of the National Academy of Science*, **8**, 115, 1922.

several hundred times the sun's mass makes the density of the order of 10^{-19} gm/cm^3.

Spectral bands of the observed character might, therefore, perhaps be produced by meteoric infall, in the very exceptional case of a star which had just entered a dark nebula of unusual density, and received its full quota of infalling material, without being far enough in to have its light seriously obscured. The bands, however, "are prevalent throughout the early stellar types, differing in intensity and extent from one star to another." They are strongest "for stars in heavily nebulous regions";[1] but the considerations here presented show that, if of meteoric origin, they should be confined exclusively to stars in such regions.

Two additional factors require to be considered. The value of L diminishes when the ratio x_0 of the star's radius to the distance at which volatilization takes place is small. The evaporation temperature E of meteoric material is certainly below 3000° and probably as low as 2500°, as is shown by electric-furnace spectra, while the constant B for a star of class A is about 7500°. Hence, by (19) x_0 is of the order of 0.1. Table II shows that this will not diminish L seriously.

A more serious difficulty has been pointed out by Shapley. Nebulosity composed of gases would in general be repelled from stars by radiation pressure. If the gas were formed by volatilization of meteors close to the stars, this difficulty is partly removed. But Table I shows, nevertheless, that when the radiation pressure considerably exceeds gravity, the depletion is greatly diminished. By hypothesis, the atoms or molecules which produce the observed depletion have ultimate lines (or bands) in the observed region. Such atoms (as the case of Ca^+ illustrates) would be repelled even by the sun, if it were not that Fraunhofer absorption in the deeper layers depletes the spectrum in just the right places. For A stars the ratio of radiation pressure to gravity, or of luminosity to mass, is ten times greater than for the sun, or more; and almost the full normal intensity of the star's radiation must be effective.

This makes it very improbable that bands of the observed strength can be of meteoric origin even for stars in nebulous regions.

[1] Shapley and Payne, *op. cit.*, p. 5.

Their presence in stars like Vega, which show no signs of associated nebulosity, offers insuperable difficulties for the meteoric explanation, and it would appear that this theory, attractive as it is, is not tenable.

It seems improbable, also, that early type stars, even in dense nebulosity, can receive sufficient infalling meteoric material to compensate for the loss of mass by radiation. Where there is no radiation pressure, the rate of infall is proportional to MR. Conditions are thus more favorable for early type stars than in the case of the depletion; but even so the gain on account of these factors is not adequate to meet the much greater rate of loss of mass by radiation. All the meteoric gas will be kept away from the star by radiation pressure, provided that $K > B^2/(B^2-E^2)$. For an A star this will certainly happen if the radiation pressure exceeds gravity by 15 per cent; for the sun the excess should be four or five times as great.

Atoms of the metals should therefore stand no chance of falling into the sun, unless they become highly ionized, and absorb only in the extreme ultra-violet (e.g., Na^+, Ca^{++}). Atoms or molecules of the permanent gases, which absorb only beyond λ 1800, should enter the sun freely, but find some difficulty with an A star.

During the whole time of their approach to the star, however, the atoms are liable to photo-electric ionization; and, once ionized, the chances of their neutralization are negligible. It is probable, therefore, that a considerable fraction of them—if not the majority—would get into a higher state of ionization, in which they were subject to but small radiation pressure. Electrons enough to neutralize their charges will, of course, be dragged in with them by electrostatic attraction.

If this happens, the arguments previously made against the meteoric origin of the observed spectral depletion are increased in force. If the radiation pressure should thus be effectively neutralized, and the whole amount of 60 tons per second fall into the sun, the average number of atoms falling into a square centimeter per second would be thirty millions (taking the mean atomic weight as 20). The whole kinetic energy which they carry in amounts to 2 ergs/cm² sec. as against 6.2×10^{10} ergs/cm² sec. of photospheric radia-

tion. It is doubtful whether this can produce a perceptible effect, even indirectly.

To compensate for the radiative loss of mass, the density of the meteoric matter must be 7×10^4 times as great, for a star like the sun, which could only happen if it were immersed in very dense nebulosity. Even in this case, the kinetic energy delivered to the atmosphere by the falling atoms would be only 1/500,000 of that flowing through from the depths. No influence on the spectrum is to be anticipated beyond the possible presence of a few emission lines demanding high excitation.

For giant stars, a still greater nebular density would be required to balance the radiative loss. For the fainter dwarfs, a much smaller density would suffice, and it is not impossible that such stars may have their evolution somewhat delayed by accretion of mass, if their paths lead them through nebulous regions. The time of a star's passage through a dark nebula, however, is so small in comparison with the probable life of a star—and especially of a faint dwarf—that frequently repeated passages would be required to produce an effect of any importance.

PRINCETON UNIVERSITY OBSERVATORY
October 20, 1928

MINOR CONTRIBUTIONS AND NOTES

MEASURES OF RADIAL VELOCITY OF α CORONA BOREALIS

In a recent[1] note on "Measures of the Variable Radial Velocity of α Corona Borealis" Professor Hartmann mentions certain measures which I made of his spectrograms of this star. As these measures may be of interest in themselves as well as indicative of the range which may be expected from measures of the same plates by different observers they are presented here in Table I.

TABLE I

Plate	Date	J.D.	Red. to Sun	V	
				Fox	Hartmann
I 597	24 Aug. 1903	2416351.30	−20.0	−18.0	− 8.7
605	22 Sept. 1903	2416380.25	−14.4	−17.3	−11.2
618	25 Sept. 1903	2416383.24	−13.6	−29.6	−37.8
624	29 Sept. 1903	2416387.24	−12.1	+ 1.9	− 3.3
625	30 Sept. 1903	2416388.24	−12.1	+17.9	+20.4
629	1 Oct. 1903	2416389.23	−11.7	+34.3	+36.0
648	19 June 1905	2417016.41	−15.2	+16.8	+20.4
650	20 June 1905	2417017.40	−15.4	+11.6	+10.1
655	27 June 1905	2417024.35	−17.1	−27.1	−23.4
666	30 June 1905	2417027.34	−17.6	−34.6	−25.9
702	27 Mar. 1906	2417297.46	+12.6	+ 0.6	+ 4.5
718	1 Apr. 1906	2417302.39	+11.2	−27.8	−25.9

These measures were made in 1906 under Professor Hartmann's direction at the Astrophysical Observatory of Potsdam. Inasmuch as I have never publicly acknowledged my obligation to him for his many kindnesses of that year I take this belated opportunity to do so.

PHILIP FOX

DEARBORN OBSERVATORY
EVANSTON, ILLINOIS
May 25, 1928

[1] *Astrophysical Journal*, 67, 270, 1928.

SPECIAL NOTICE TO CONTRIBUTORS CONCERNING CUTS

Owing to the increasing trouble and expense of preserving the blocks of halftones and zinc etchings used in this *Journal*, the management of the University of Chicago Press has found it necessary to adopt a policy of destroying such blocks as soon as the number of the *Journal* containing them has been issued and all reprints ordered in advance have been struck off. Those who appreciate the pains taken in the editorial office in the preparation of our illustrations will realize that such action, however necessary, may seem like ruthless efficiency. The managing editor has therefore secured from the publishers a mitigation of this sentence, in the belief that authors might desire to preserve for future use blocks of engravings illustrating their own articles. The University of Chicago Press therefore announces that hereafter authors may secure such cuts by filling out the blank form sent with the galley proofs, which provides also an opportunity for ordering reprints. The blocks will be sent for the nominal cost of handling, packing, and of postage, at the rate of twenty-five cents per cut, sent with the order. If reprints are ordered, the cost of cuts may be added to the bill for reprints and combined therewith at the office of the Press.—THE EDITORS.

January 1929

ERRATA

Vol. **68,** No. 4, November, 1928, article on "Stellar Radiation Measurements," by Edison Pettit and Seth B. Nicholson:

Page 279, third line from bottom, *for* "radiometric" *read* "visual."

Page 285, fourteenth line from top, *for* "four-tenths" *read* "ten-fourths."

Page 301, fourth line from bottom, *for* "radiometric" *read* "visual."

REVIEWS

A Guide to the Constellations. By SAMUEL G. BARTON AND WM. H. BARTON. New York: McGraw-Hill Book Co., 1928. Pp. x+74. Charts 17. Figs. 5. $2.50.

There seems to be an increasing interest on the part of the general public in astronomy, particularly in that portion which is descriptive in character. To such, this star atlas will be of great service.

The chief feature of the book is found in the excellent set of thirteen charts of the constellations, with the stars in white on a blue background. The size of the page, 9×12 inches, permits a good scale to be used. The constellations are outlined and named, but a serious omission is made in not printing on each chart the names of stars of the first magnitude. This could easily be done without crowding the chart. These names are not even given on the four additional detail charts. On the other hand, the lines joining the stars seem to be overdone and tend to confuse rather than help the beginner.

A valuable feature of the book is the page of descriptive matter accompanying each chart and giving the chief facts of interest about the stars and nebulae found therein. Elsewhere are given the leading facts about the sun, moon, and earth, and some general information about the other planets. However, in this connection, it would seem that mention should be made of some of the physical characteristics of the planets, such as the belts of Jupiter or the rings of Saturn. Since three pages are given to the mythology of the constellations, it would appear that at least one page should be assigned to such description.

All students of astronomy will find a convenience in the collection of facts in the tables which also include the pronunciation of the names of the constellations and principal stars. The latter will be much appreciated by those who have found many of the names difficult to pronounce.

The publishers are to be commended on the attractive appearance of the book as to paper, type, and binding.

W. H. GARRETT

Annual Tables of Constants and Numerical Data. Issued by the PERMANENT INTERNATIONAL COMMITTEE (C. MARIE, general secretary). Vol. V, Part II, for the years 1917–1921; Vol. VI, Parts I and II, for the years 1923–1924. Paris: Gauthier-Villar, 1926–1927–1928. New York and Chicago: McGraw-Hill Book Co., American Agents. Quarto. Pp. 1–904; 905–1931; 1–679; 680–1675. Vol. V, $20; Vol. VI, $25, bound.

These portly tomes form the continuation of the important series begun in 1910 under the auspices of the Union of Pure and Applied Chemistry and of the International Association of Academies. The course of events has changed the patronage of the work as the volumes have been issued, but the plan has continued as first formulated with some additions, such as the inclusion of biology. The permanent committee of editors for the volumes under review includes Professors Ernst Cohen, P. Dutoit, A. Egerton, O. Scarpa, and E. W. Washburn, and their names are a sufficient guaranty for the wisdom displayed in the selection of tables to be included, under the able editorial care of Dr. C. Marie, of Paris.

We note as of particular interest to astrophysicists the following chapters: "Spectroscopy"; "Cosmic Physics"; "Atomic Weights"; "Molecular Weights"; "Photography"; "Radiation"; "Luminescence"; "Photometry"; "Optics"; "Radioactivity"; "Ionisation, Electrons, X-Rays"; and "Geophysics."

The material has been collected from the available standard sources in the proceedings of learned societies and the technical journals of science. The tabular matter is of course very extensive, and is printed in as condensed a form as possible. The chances for typographical errors are of course very great, but they have undoubtedly been kept at a minimum and lists of errata have been published by the editors.

The portions dealing with spectroscopy from Volumes V and VI have been reprinted as separate volumes, and will be reviewed as such in this *Journal*. An increasing use of diagrams is seen in Volume VI.

The typography is adequate, and the paper is probably of sufficient permanence.

These *Annual Tables* should not be confused with the *International Critical Tables*, prepared under the auspices of the International Research Council, and of the National Academy of Sciences, by the National Research Council of the United States, which secured the large funds necessary for the publication. There has been an effective co-operation in the preparation of these separate and complementary sources of numerical data, and for this purpose there is a certain interlocking of the

editorial boards. The *Critical Tables* for the earlier years are to a considerable extent based upon the first volumes of the *Annual Tables*, but also include other sources. Five volumes of the *Critical Tables* have been issued, and it has been found necessary to prepare two additional volumes, which are expected to appear in 1929 and 1930.[1]

We understand that it is the plan to continue the publication of the *Annual Tables* through ten volumes (the last covering the year 1929), although the heavy cost must be a serious burden to the editors and publishers.

An Index to Volumes I–V of the *Annual Tables* is promised for publication in 1929.

Well-equipped libraries of larger scientific institutions will find it desirable to maintain a complete file of both of these encyclopedic series of contemporary records of progress in the exact sciences.

Reziprokentafel aller ganzen Zahlen von 1 bis 10000. By M. VAN HAAFTEN. Groningen: P. Noordhoff, 1926. 25×16 cm. Pp. 50. Bound, 2.40 florins.

This is a very convenient table of reciprocals, in the form of single entry with four double columns to the page, covering two hundred numbers. The numbers are given in modern numerals, of clear large type; the reciprocals in old-style numerals, to seven decimals. No decimal point is given for the reciprocals, which seems to us a defect.

In view of the existence of the well-seasoned Barlow's tables, and of Moses Cotsworth's *Direct Calculator* (1903), as well as of Oakes's tables, entitled *Table of the Reciprocals of Numbers from 1 to 100,000*, the issue of another table may seem superfluous. Nevertheless, the freedom from eyestrain presented in the volume under review and its convenient arrangement recommend it as a valuable table to have within reach in the computing-room or office.

[1] The price of the *Critical Tables* is $12.00 per volume, and the McGraw-Hill Book Company also act as publishers, in behalf of the National Research Council.